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Methods for spherical data analysis and visualization

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Abstract

A systematic analysis of the localization of objects in extra-personal space requires a three-dimensional method of documenting location. In auditory localization studies the location of a sound source is often reduced to a directional vector with constant magnitude with respect to the observer, data being plotted on a unit sphere with the observer at the origin. This is an attractive form of data representation as the relevant spherical statistical and graphical methods are well described. In this paper we collect together a set of spherical plotting and statistical procedures to visualize and summarize these data. We describe methods for visualizing auditory localization data without assuming that the principal components of the data are aligned with the coordinate system. As a means of comparing experimental techniques and having a common set of data for the verification of spherical statistics, the software (implemented in MATLAB) and database described in this paper have been placed in the public domain. Although originally intended for the visualization and summarization of auditory psychophysical data, these routines are sufficiently general to be applied in other situations involving spherical data. © 1998 Elsevier Science B.V. All rights reserved.

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1. Introduction

While science is arguably based upon observation statement, even relatively unsophisticated analysis of simple data involves assumptions. This is often only implicit in the analytical or statistical methods employed in analysis. Another subtle form of analytical assumption is often buried in the methodology employed in visualizing the data. Although graphical data representation has a long history (Tufte, 1990), the growing availability of inexpensive and powerful computers has resulted in renewed interest in data visualization as a form of analysis for complex or large data sets. One area in neuroscience where data sets can be both large and, from an analytical perspective, fairly complex, is in the representation of extra-personal space and how an animal or individual relates to that space in some meaningful way.

In our laboratory we have been examining how the mammalian nervous system processes auditory information to generate a neural representation of space that results in our perceptions of the auditory world. In its simplest form, we refer to the location of an auditory object (a sound source) in terms of its direction and distance from the observer. As the observer occupies a point in space, the representation and analysis of these data involve at least three spatial dimensions and then a number of other dimensions related to the nature of the signal (e.g. frequency and time) and the experimental manipulation (the independent variables). The most common form of experiment is the placement of a sound source at an unseen location in space and an indication by the subject of the location of the target. The disparity between the indicated and actual location of the auditory target is used as a measure of the localization accuracy of the subject, and we use the

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term 'error' in this paper to describe this value. The analysis of these data generally concentrates on the directional vector of the auditory object and, in most cases, ignores the distance effects. Under this assumption the representation of these data reduces to a more tractable two-dimensional spherical display of the data. Furthermore, the plotting and manipulation of spherical data in real time makes this form of analysis attractive since the complex spatial relations in the data can be easily apprehended as the viewpoint is rotated. Indeed, this is one of the principal advantages of visualization of complex data sets.

Previously, there has been a range of methods applied to the summary description and statistical analysis of these data. For example, Oldfield and Parker (1984) used simple $X-Y$ plots of error verses target position and shaded contour plots of the errors on an azimuth/elevation grid. Wightman and Kistler (1989) used $X-Y$ source position versus perceived position plots; and Makous and Middlebrooks (1990) used ellipses drawn on a double pole spherical plot where the size of the major and minor axes were proportional to the signed errors in the vertical or horizontal directions. All of these representations involve separate analysis of the azimuth and elevation components of the data, such as calculating the variance of the azimuth and elevation localization errors for each target location in space.

This approach does not account for some features of these data since azimuth and elevation are likely to covary. We have applied the Kent distribution (Kent, 1982) instead of the commonly used Fisher distribution (described in Fisher et al., 1993) to analyze these spherical data. The Fisher distribution assumes that the data is rotationally symmetric whereas the Kent distribution can be used to model asymmetric data. Such an approach is more likely to expose the coordinate system used by the auditory central nervous system to represent auditory extra-personal space. This is an important methodological step as it allows the comparison of localization performance in individuals or groups to be compared with the spatial variation in their auditory spatial cues to a sound's location. Such an approach provides insights into the processing strategies employed by the auditory system in computing and representing the spatial locations of sound sources (see Carlile, 1996, for review). In considering these issues we have also sought to apply a number of robust statistical methods to the auditory localization data collected in our laboratory and to combine these with a convenient set of visualization tools. We have collected together many of the relevant statistical methods and describe here a library of data manipulation, plotting, summary statistical procedures and routines for hypothesis testing using spherical data. These methods have been developed using MATLAB (The MathWorks, Inc.), a popular data analysis and visualization package and have now been made available as public domain software. This small library, called Spak (for 'spherical package'), provides a flexible set of tools for manipulating and processing spherical data (Spak is freely available upon request from the authors, or via the World Wide Web site http://www.physiol.usyd.edu.au/simonc/). It is hoped that, by providing a common resource for the analysis and interpretation of these complex data sets, a greater consistency in approach will be encouraged and thus facilitate more rigorous comparisons between studies in this area. Although these routines were developed to serve the requirements of the research community examining auditory localization, the routines are sufficiently general that they could be employed in other research areas using spherical data (e.g. astronomy, geodesy, geology, geophysics and mathematics; see Fisher et al., 1993).

2. Methods

2.1. Spherical coordinate system

The quantitative description of spherical data is dependent on the definition of a particular spherical coordinate system. A number of systems are in general use (Fisher et al., 1993). The two most common in the auditory literature are a single pole system (analogous to the planetary coordinate system), and a double pole system which shares the longitudinal circles of the single pole system (denoting the elevation of a source) but has an orthogonal series of circles centered on the interaural axis. As these two systems share the nomenclature of azimuth and elevation, it is essential that, when reference is made to spherical data, the spherical coordinate system is always explicit. Unhappily, this has not always been the case in the auditory literature and has led to some confusions. We have selected the single pole system since it is more intuitive (see Carlile, 1996, for discussion).

A two-dimensional, single pole spherical coordinate system was used to describe points on a unit sphere centered about the subject's head. A point directly in front corresponds to zero azimuth and zero elevation. The azimuth coordinate (az) increases in a clockwise direction from the subject's head and elevation (el) increases in an upwards direction (see Fig. 1). This coordinate system will be referred to as 'hoop coordinates' since it corresponds to the coordinates used in our laboratory by the automated robot arm in placing the auditory stimulus on an imaginary sphere surrounding the subject. This coordinate system is commonly used in auditory localization literature and positions on the sphere are represented as the ordered pair (az, el). It should be noted that this particular choice of coordinate system restricts the analysis to data lying on the

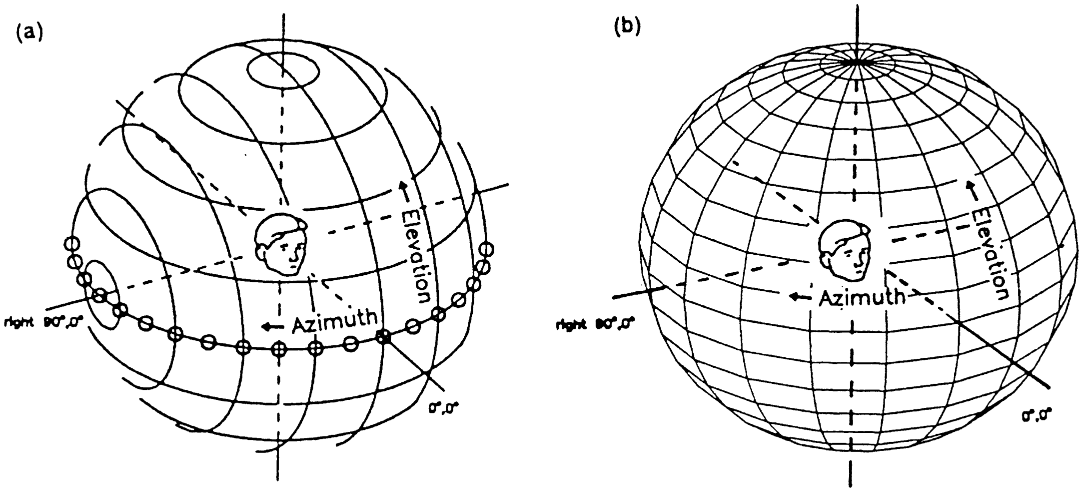

Fig. 1. The double pole coordinate system (a) compared with the (single pole) hoop coordinate system (b).

sphere, and in particular, does not give a measure of distance (which would be the third dimension). One way in which this method could be used to display distance data would be to use surface color to indicate distance, however, this is not explored here.

Although, by convention, hoop coordinates are always used in our software to describe positions in space, it is often mathematically more convenient to perform calculations in polar coordinates (θ, ϕ). To convert between the coordinates, the following formulae were used:

$$\theta = 90 - \text{el} \qquad \phi = -\text{az}$$

2.2. Mean direction

In order to calculate the mean direction of a set of points on the sphere, it is not sufficient to average the azimuth and elevation values since the hoop coordinate system is discontinuous. A simple averaging of points at hoop coordinates (0, 0) and (359, 0) would give the undesirable value of (179.5, 0).

Instead, the mean direction $(\bar{\theta}, \bar{\phi})$ of a set of n data points P_i $(=(\theta_i, \phi_i))$ was computed by firstly finding the Cartesian coordinates (also known as the direction cosines) (x_i, y_i, z_i) using the formula (Fisher et al., 1993):

$$x_i = \sin\theta_i \cos\phi_i, \qquad y_i = \sin\theta_i \sin\phi_i, \qquad z_i = \cos\theta_i$$

The vector sum of the unit vectors OP_i (O is the origin) is then computed:

$$S_x = \sum_{i=1}^{n} x_i, \qquad S_y = \sum_{i-1}^{n} y_i, \qquad S_z = \sum_{i=1}^{n} z_i$$

The mean direction has a resultant length of:

$$R = \sqrt{S_x^2 + S_y^2 + S_z^2}$$

Since our spherical coordinate system only allows unit length vectors, the resultant length can be used as a measure of dispersion (Wightman and Kistler, 1989). R can range between 0 and n, with a large value corresponding to low dispersion, and small values corresponding to increasingly uniform distributions of the data on the sphere.

For the direction cosines:

$$(\bar{x}, \bar{y}, \bar{z}) = (S_x/R, S_y/R, S_z/R)$$

These can be converted into polar coordinates using the following formulae:

$$\theta = \arccos(\bar{z}), \qquad \phi = \arctan(\bar{y}/\bar{x})$$

2.3. Kent distribution

Localization errors can be analyzed using the Kent distribution (Kent, 1982, Fisher et al., 1993)[1] which can deal with asymmetric data. Using this method of modeling, no assumptions are made about the distribution of the data. The Kent distribution is a generalization of the Fisher distribution which assumes that the data is unimodal and has rotational symmetry. As can be seen in Fig. 2 (extracted from the same data set as Fig. 3), the top set of data points has a much wider variance in the azimuth direction thus leading to an ellipse shape (so is best described using the Kent distribution), whereas the bottom set of data is much more circular since the data is almost rotationally symmetric and can be described by a Fisher distribution.

[1] The 1987 edition of Fisher et al. had some errors in the sections detailing the computation of the Kent distributuion. The 1993 paperback edition corrected these (some in the text and some in the errata). We have verified with the authors that there is a final minor mistake in Example 5.28, but such elliptical confidence cones are not used in our software. Instead, we use the standard deviation which is proportional in size to the elliptical confidence cone.

The Kent distribution is described by the parameters **G**, κ and β, where **G** is a 3×3 matrix containing the three 3×1 column vectors (ξ_1, ξ_2, ξ_3). ξ_1 is the mean direction of the distribution, ξ_2 is the direction in which the data density is the highest (major axis), and ξ_3 is the direction of least data density (minor axis). It is most convenient to think of **G** as being the rotation matrix which best aligns the mean direction to the 'north pole' (i.e. (0, 0, 1) in Cartesian coordinates) of the sphere, with the principal components aligned with the (θ, ϕ) axes, the 'principal components' of a dataset being the set of M orthogonal vectors in that space that account for the maximum amount of variance in the data; found using principal component analysis—or the Karhunen–Loève transform as it is otherwise known (see e.g. Jolliffe, 1986). The κ parameter describes the degree of concentration of the data about the pole of the distribution, and β is an ovalness parameter which is small for circular data and increases as the data becomes more ovoid. Using the shape parameters κ and β, unimodal distributions, $(\kappa/\beta)\geq 2$, can be distinguished from bimodal distributions, $(\kappa/\beta)<2$.

Estimates of the parameters of the Kent distribution (Fisher et al., 1993) can be computed for n data points (θ_i, ϕ_i) in polar coordinates (with corresponding direction cosines (x_i, y_i, z_i), resultant length R and mean direction ($\bar{\theta}$, $\bar{\phi}$) using the following formulae:

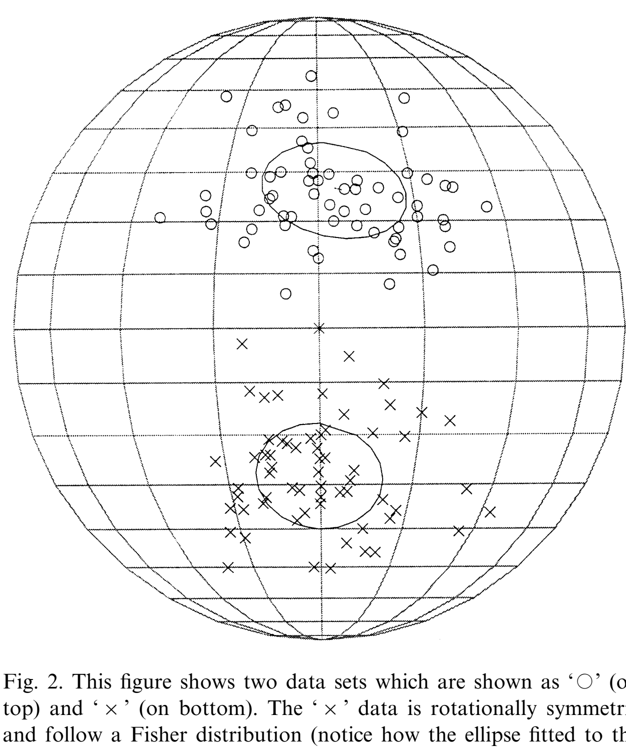

Fig. 2. This figure shows two data sets which are shown as '○' (on top) and '×' (on bottom). The '×' data is rotationally symmetric and follow a Fisher distribution (notice how the ellipse fitted to the data is circular). The '○' data is better modeled by the more general Kent distribution. The Kent distribution need not be rotationally symmetric and the major and minor axes of the ellipses drawn are aligned with the directions of greatest variance in the data.

$$H=\begin{pmatrix}\cos\bar{\theta}\cos\bar{\phi} & \cos\bar{\theta}\sin\bar{\phi} & -\sin\bar{\theta}\\ -\sin\bar{\phi} & \cos\bar{\phi} & 0\\ \sin\bar{\theta}\cos\bar{\phi} & \sin\bar{\theta}\cos\bar{\phi} & \cos\bar{\theta}\end{pmatrix}$$

$$T=\begin{pmatrix}\sum x_i^2 & \sum x_iy_i & \sum x_iz_i\\ \sum x_iy_i & \sum y_i^2 & \sum y_iz_i\\ \sum x_iz_i & \sum y_iz_i & \sum z_i^2\end{pmatrix}$$

$$\mathbf{B}=\mathbf{H}'(\mathbf{T}/n)\mathbf{H}$$

If the elements of **B** are represented as:

$$B=\begin{pmatrix}b_{11} & b_{12} & b_{13}\\ b_{21} & b_{22} & b_{23}\\ b_{31} & b_{32} & b_{33}\end{pmatrix}$$

then we define $\psi=\frac{1}{2}\arctan[2b_{12}/(b_{12}-b_{22})]$ and compute the rotation matrix:

$$G=H\begin{pmatrix}\cos\psi & -\sin\psi & 0\\ \sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{pmatrix}$$

The matrix **V**, calculated by:

$$\mathbf{V}=\mathbf{G}'(\mathbf{T}/n)\mathbf{G}$$

has elements:

$$V=\begin{pmatrix}v_{11} & v_{12} & v_{13}\\ v_{21} & v_{22} & v_{23}\\ v_{31} & v_{32} & v_{33}\end{pmatrix}$$

and if we define $Q=v_{11}-v_{22}$, using this and the resultant length, we can calculate the shape parameters:

$$\kappa=\frac{1}{2-2R-Q}+\frac{1}{2-2R+Q}$$

$$\beta=\frac{1}{2}\left(\frac{1}{2-2R-Q}-\frac{1}{2-2R+Q}\right)$$

The parameters $(\mathbf{G},\kappa,\beta)$ describe the Kent distribution.

The Kent distribution can be used to determine whether a sample comes from a Fisher distribution as opposed to a Kent distribution using the following test statistic:

$$K=\frac{n(\frac{1}{2}\kappa)^2 I_{1/2}(\kappa)Q^2}{I_{5/2}(\kappa)}$$

where $I_{1/2}(\kappa)$ and $I_{5/2}(\kappa)$ are modified Bessel functions of the first kind. The hypothesis that the data comes from a Fisher distribution rather than a Kent distribution is rejected at the $100\alpha\%$ level if $K>-$

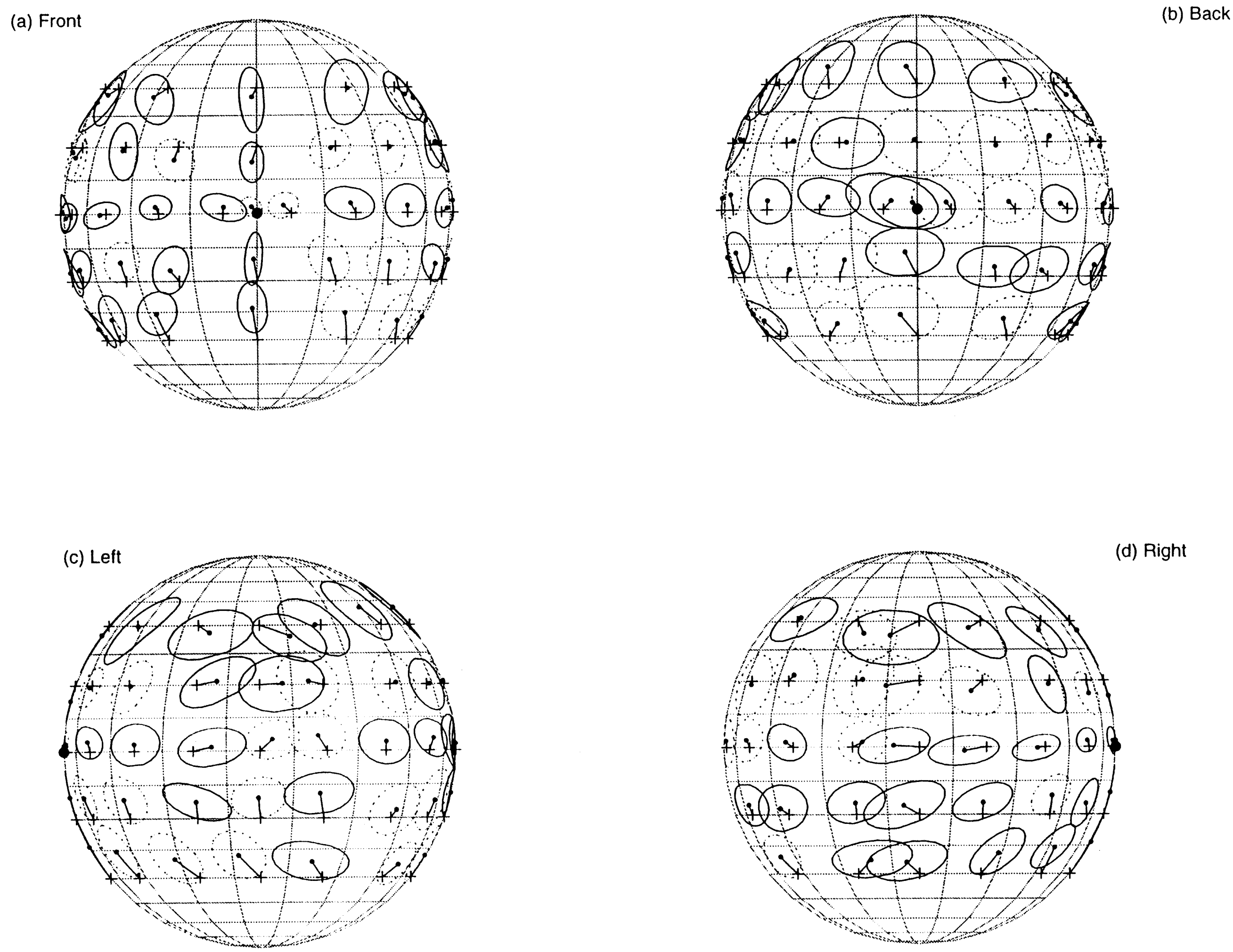

Fig. 3. Spherical plot showing major and minor axes of the data variance obtained using the Kent distribution.

$2 \log \alpha$. For the example shown in Fig. 2, $K = 9.1$ for the top (Kent) example and $K = 0.4$ for the bottom (Fisher) example.

In order to obtain the ellipses displayed in Fig. 2 and Fig. 3, the data were first rotated using the **G** matrix, i.e.:

$$\begin{pmatrix} x'_i \\ y'_i \\ z'_i \end{pmatrix} = \boldsymbol{G} \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix}$$

This procedure aligns the principal components of the data with the azimuth and elevation axes, centered about the pole. The standard deviations along the axes are then calculated and an ellipse about the North pole with major and minor axes one standard deviation in size is computed. The ellipse is then rotated back to the mean position using the **G** matrix to produce the plotting coordinates of an ellipse centered about the mean direction with major and minor axes in the principal directions of data variance.

2.4. Data collection

The aim of these types of experiments is to examine the accuracy with which a subject can determine the location of an auditory target or sound source. In our laboratory the target is placed randomly on the surface of an imaginary sphere (1 m radius) centered on the subject's head using a computer controlled robot arm (see Carlile et al., 1996). The experiment is carried out in a darkened anechoic chamber to avoid extraneous acoustic and visual cues to target location. Following a short (150 ms) auditory stimulus, the subject is asked to turn to face the source of the sound and point to it with their nose. The position of the head is determined using an electromagnetic tracking device which provides azimuth, elevation and roll as well as the translational position of the head (Polhemus, Isotrack). Following appropriate training (Carlile et al., 1996) this provides a reliable and objective measure of the perceived location of the sound. Overall localization accuracy and the types of mislocalizations evident under different listening conditions and with different auditory targets can be used to probe localization processing.

2.5. Data manipulation

Although the extraction of data is a relatively trivial task, we have found that a large percentage of our previous code involved data manipulation. The simple interfaces, along with a small set of routines to sort, select and extract the data have considerably reduced the complexity of the scripts. As illustrated in Section 3, very small programs using this library are capable of implementing complicated procedures.

2.6. Library interface routines

In this section, the Spak routines available to the user are described. Conventions that are followed in Spak are that all data selection routines have an '_ld' (localization data) suffix, all spherical routines (statistical and plotting) have a '_sp' (spherical) suffix, and all conversion routines have a '2' in them (for example, *cart2hp* converts from Cartesian to hoop coordinates. For all of the main library routines (those that have '_ld' or '_sp' suffixes), coordinates are expressed in hoop coordinates (as described above).

The localization data are assumed to be arranged as an $n \times 4$ matrix where n is the number of points in the data set and the four columns correspond to source azimuth, source elevation, and the azimuth and elevation of the localization estimate, respectively. Typically, data is extracted and processed one record at a time, where a record is the $k \times 4$ matrix representing all recordings from a given source location (i.e. all vectors in the matrix have identical first and second columns).

A summary of the commands available in Spak is given in Table 1. A very small set of flexible library routines provides great utility.

3. Results

In this section, we provide examples of the types of visualizations possible using Spak. A feature of this software is that very few lines of code are required to produce visualizations of auditory localization data. All of the code used to generate the plots in Figs. 3–5 are given in Appendix A.

3.1. Front–back confusion plots

There are basically two different types of auditory localization errors. The first type, called 'local errors', are those where the subjects perceive the location to be within a few tens of degrees of the actual location. The second type, where the subject correctly identifies the azimuth angle of the target with respect to the median plane but makes an error in the hemisphere that the sound is judged to be located. These are called 'front–back confusion errors', as, for example, where a sound is located 10 degrees to the left of the frontal median plane and the subject perceives the location to be say 15 degrees left of the rearward median plane. In our applications, the latter type of error is relatively infrequent (at most a few percent of the total number of localization judgments in a given study), and results in a weakly bimodal data set (the Kent distribution analysis routines also check and report on whether the data is unimodal or bimodal). One approach is to extract the front–back confusion data (e.g. Makous and Middlebrooks, 1990) and analyze these data separately (see Carlile, 1996, for details). The set of routines described here provide a filter to achieve this (rfb_sp( )). Other methods used for dealing with front–back confusions include resolving front–back confusions by reflecting the perceived location about the interaural axis before computing descriptive statistics (e.g. Wightman and Kistler, 1989) or avoid summary statistics altogether by showing all of the data graphically (e.g. Kistler and Wightman, 1992). In the latter two cases, routines from

Table 1
Summary of the routines available in Spak

Routine	Description
sel_ld( )	Returns all of the records bound within the rectangle defined by two locations on the sphere
sort_ld( )	Arranges the data in a canonical form (increasing in elevation with the azimuth increasing within the same elevation) so that all data from the same source location are adjacent
next_ld( )	Returns the next record as well as a copy of the matrix with the chosen record removed. The truncated matrix can be used as the next argument to next_ld( ) to iterate through all of the data
get_ld( )	Returns the data associated with a particular location
closest_ld( )	Returns the record with the closest match (Euclidean distance) to a particular location
rfb_ld( )	Remove front–back confusions associated with localization data
amov_sp( )	Translate the points specified by the second parameter by the amount specified in the first parameter
mean_sp( )	Computes the centroid of a record
median_sp( )	Computes the median of a record
kent_sp( )	Compute the Kent distribution parameters of a record
draw_sp( )	Draw a sphere
line_sp( )	Draw a polygon which connects the points given as input arguments
plot_sp( )	Draw dots at each point given in the input matrix

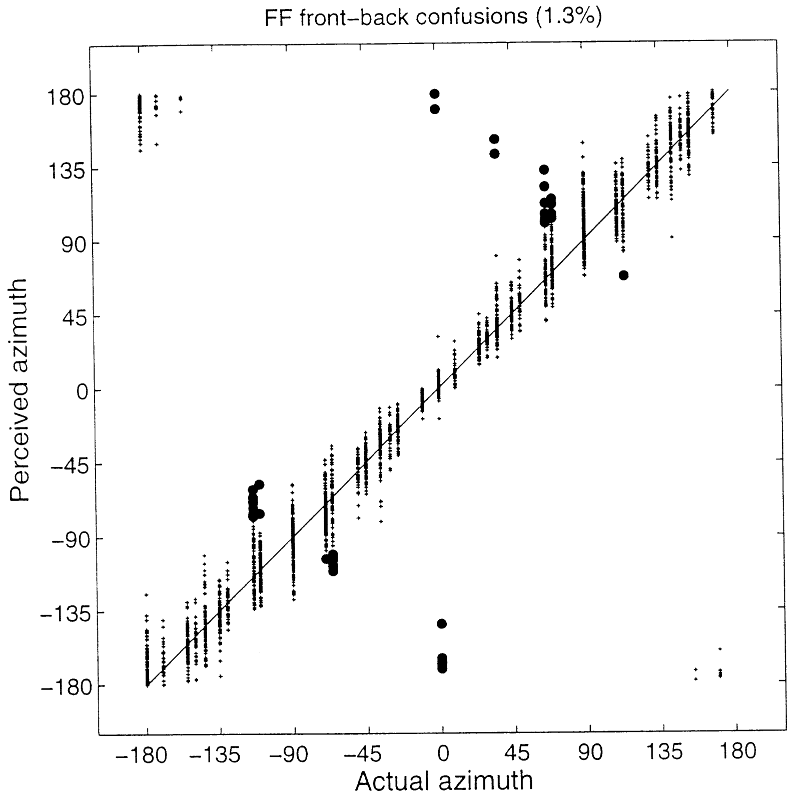

Fig. 4. Plot showing front–back confusion information on an $X-Y$ plot.

the Spak library could be employed along with standard MATLAB functions to filter the data appropriately before plotting.

The front–back confusions can be visualized using plots of Fig. 4 (Makous and Middlebrooks, 1990), the file 'testfb.m' in Appendix A showing the code used to produce the plot. The code uses the sample psychophysical data contained in the file 'bigloc.asc' and the processing shows the results of extracting out the front–back confusion data in the data set and plotting these on an $X-Y$ plot where elevation has been collapsed. The extracted data is shown by the filled circles and the remaining data is plotted as small crosses. Lines 6 and 9 extract the local errors and the front–back confusion data, respectively, which are plotted on lines 12 and 14. The other lines of code are used to load the data and to annotate the plot, and are standard MATLAB commands.

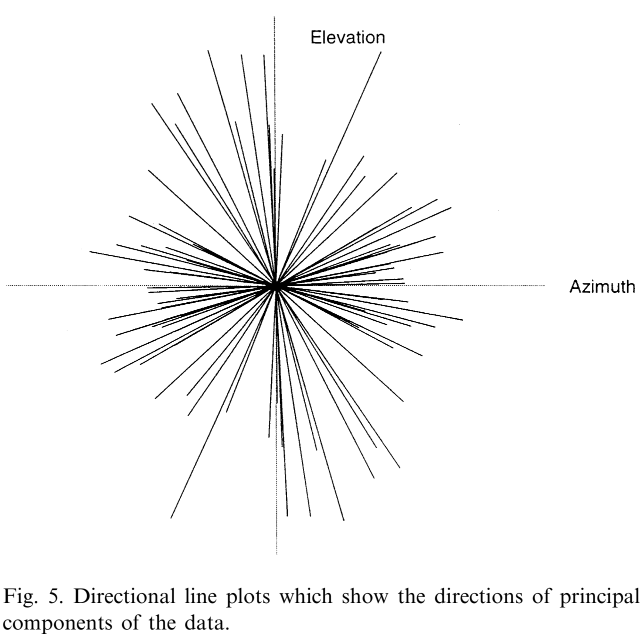

Fig. 5. Directional line plots which show the directions of principal components of the data.

3.2. Directional line plots

Directional line plots can be used to visualize the directions of the principal components of the errors. The file 'testdl.m' in Appendix A (line 24) produces the plot of Fig. 5. The kent_sp( ) routine is called to compute the direction of the principal components (line 43), and then a line through the origin, which points in the direction of the first principal component and has length proportional to the standard deviation is computed using the amov_sp( ) routine (line 46) and plotted on line 47. Lines 38–40 and 50–51 perform a typical loop which iterates through all the data, one location at a time, and this type of loop is explained in greater detail in the next section.

On the resulting plot, if the errors are aligned with the coordinate system (as has been generally assumed in previous analyses), they will appear predominantly aligned with the vertical and horizontal axes, but in the case of the example in Fig. 5, this is clearly not the case.

3.3. Spherical plots showing kent distribution

As a final example of the analysis and visualization possible using Spak, the MATLAB program which is shown in file 'testkent.m' of Appendix A is described in this section, producing the plot shown in Fig. 3. The program analyses each record using the kent_sp( ) routine, and makes an ellipse centered around the centroid (i.e. the mean direction) of the measured results. A line is drawn between the centroid and the source location, and an ellipse with major and minor axes aligned with the principal components and with a size of one standard deviation on each axis is drawn. If the data fits a Fisher distribution (i.e. it is rotationally symmetric in nature), this ellipse is drawn with a dotted line, otherwise, it must be a Kent distribution and it is drawn in a solid line.

On line 57, the matrix which describes the experimental data is loaded. This is typical of the data resulting from a series of psychophysical experiments. The subroutine akent( ) is called with arguments describing the location of the data of interest, and all of the processing is done in this routine. Note that, to do the 'back' plot of Fig. 3, akent( ) needs to be called twice (lines 67 and 68) to account for the fact that our coordinate system is discontinuous for this projection and we must therefore do the left and then the right projections separately. The akent( ) routine, which starts on line 76, begins by using sel_ld( ) to remove the data which will not be used for a particular projection. The data is then sorted to put it into a canonical form (line 81). Each record is then extracted using next_ld( ) (lines 86 and 98) which puts the record in 'dat' and the truncated data set in 'sloc'.

The routine kent_sp( ) does all of the statistical work used in this example. This routine is called with the experimental data only (extracted using the MATLAB expression dat(:,[3 4])), and returns G, kappa, beta, q, ellz, ell, ln and isk. G, kappa, beta and q correspond to the Kent parameters $\mathbf{G}$, κ, β and Q described earlier, the two element vector ellz contains the sizes of one standard deviation along the principal components, ell contains a vector which if joined together forms an ellipse of size one standard deviation, and with the major and minor axes aligned with the principal directions of the data errors, ln contains a vector of points making a line from the source location to the centroid, and isk being a boolean flag which is set if the data comes from a Kent distribution and zero if it is Fisherian.

On line 60, the output is set to be in landscape mode, and the current figure is cleared on line 61. Line 63 indicates that we wish to make 4 plots arranged in a 2×2 grid, and the first graph is selected. The akent( ) routine is then called which will draw a sphere and plot the data on the first graph. The above procedure is repeated for the back, left and right plots.

The akent( ) routine firstly draws a sphere if dosp is set (line 82–84). The condition is required to avoid drawing two spheres when akent( ) is called twice for the same projection on lines 67–68. The view is then changed to that the location specified in the vcoord parameter becomes the center of that projection. Since the coordinates are assumed to be in hoop coordinates, they must first be converted to Cartesian coordinates for the MATLAB view( ) function.

Depending on the value of isk, ecol can be set to different line styles in lines 90–94. On line 95, a dot is drawn at the centroid of the data. A line is drawn from the source location to the centroid on line 96, and the ellipse is drawn with plotting parameter ecol on line 97.

The size of the ellipses in plots thus generated gives a good summary of auditory localization performance; the shape gives indications as to the relative contributions of the principal components; the orientation of the ellipse shows the alignment of the directions of greatest variance and the lines from the source locations to the centroid give an indication of the absolute errors of an auditory localization trial. We have found that this type of plot gives a very good summary of the performance of an experiment in auditory localization.

4. Discussion

This paper documents the implementation of routines for data management, visualization and the statistical description of spherical data. In an effort to maximize the utility of these routines we have a exploited a very widely used data visualization and analysis package (MATLAB). The data management features of this package facilitate the efficient manipulation of large amounts of spherical data (>104 data points). Data can be sorted and that associated with particular locations, or a range of locations can be extracted. Considerable effort has gone into implementing routines that represent a single functional step to provide the widest range of utility for each routine. Complex analytical sequences can be assembled with just a few lines of script.

A consistent set of coordinate conversion routines allows seamless movement from the popular single pole description of spatial location used in auditory research to spherical coordinates and also to the Matlab plotting coordinates. The coordinates conversion routines are integrated into plotting routines so that the user only has to operate in the 'native' single pole descriptions of spatial location. The visualization routines exploit the three-dimensional plotting capabilities of MATLAB and could be integrated into an interactive user interface that is also available with MATLAB. Although these routines have been originally written to facilitate the analysis of auditory localization data they are sufficiently generalized to be applied to any spherical data set. At most, a user may have to write small MATLAB script to convert the coordinate system used by their own data to a spherical coordinate system to make available the full functionality of this package.

As illustrated using the sample data set in Fig. 2, the distributions of localization errors for many spatial locations are not rotationally symmetrical and are better described using an ellipse. The axes of the ellipse correspond to the first two principal components and, for a significant fraction of the data, the axes are not aligned with the axis of the coordinate sphere. This is most clearly seen in the directional line plots (Fig. 5) introduced in this paper. Summary statistics computed using the directions of the principal components can lead to more revealing information than either analyzing the azimuth and elevation axes independently, or combining them and assuming rotational symmetry. Similar kinds of spherical analysis of the underlying cues to sound location will indicate to what extent the characteristics of the error distributions are dependent on the nature of the spatially dependent changes in the cues to locations.

In the previous studies of auditory localization processing at least two coordinate systems have been used in describing spatial location (single pole and double pole). A lack of a standard method for representation and analysis has resulted in enormous difficulties in comparing the results of different studies in this area. It is hoped that researchers will be able to use this package to analyze and display results from their own experiments and hence facilitate a greater uniformity in the presentation and analysis of data in this and related research fields.

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Appendix A. Sample MATLAB scripts using Spak

```
% file testfb.m

% test script for front-back confusion plots
load 'bigloc.asc' -ascii

% extract the F-B confusion errors
[res_loc,b]=rfb_ld(bigloc,10,40);

% extract them
F_B=bigloc(b,:);
%plot the confusion matrix using and X-Y plot
clg;
plot(F_B(:,1),F_B(:,3),'r.','MarkerSize',16)
hold
plot([-180 180],[-180 180],'g')
plot(res_loc(:,1),res_loc(:,3),'b+','MarkerSize',2);
axis([-210 210 -210 210])
axis('square')
ylabel('Perceived azimuth','FontSize',14)
xlabel('Actual azimuth','FontSize',14)
set(gca,'XTick',[-180 -135 -90 -45 0 45 90 135 180]);
set(gca,'YTick',[-180 -135 -90 -45 0 45 90 135 180]);
title('FF front-back confusions (1.3%)','FontSize',12);

% file testdl.m
% test script for directional line plots
load 'bigloc.asc' -ascii

% extract the F-B confusion errors
[res_loc,b]=rfb_ld(bigloc,10,40);

draw_sp(1);

%view(hp2cart([0 90]));
set(gca,'XLim',[-0.2 0.2])
set(gca,'YLim',[-0.23 0.23])
set(gca,'ZLim',[-0.13 0.13])

sloc = sort_ld(res_loc);
[dat, sloc] = next_ld(sloc);
while (length(dat) > 0)
        disp(' ')
       disp(dat(1,[1 2]))
        [G,kappa,beta,q,ellz,ell,ln,isk] = kent_sp(dat(:,[3 4]));
        disp(ellz)
        if (isk)
                rln = amov_sp(ln, ln(1,:));
                line_sp([rln(size(rln,1),:); -rln(size(rln,1),:)], 2, 'm');
                %drawnow;
        end;
        [dat, sloc] = next_ld(sloc);
end
h=text(0.1,0.25,0,'Azimuth');
h=text(0.03,0.03,0.12,'Elevation');

% file testkent.m
% test Spherical directional plots of Kent distribution
load 'bigloc.asc' -ascii

[bigloc,b]=rfb_ld(bigloc,10,40);     % extract the F-B confusion errors
orient landscape
clf;

subplot(2, 2, 1);
akent(bigloc, -90, 90, -90, 90, [0 0], 1, '(a) Front');

subplot(2, 2, 2);
akent(bigloc, -180, -90, -90, 90, [-180 0], 1, '');
akent(bigloc, 90, 180, -90, 90, [-180 0], 0, '(b) Back');

subplot(2, 2, 3);
akent(bigloc, -180, 0, -90, 90, [-90 0], 1, '(c) Left');

subplot(2, 2, 4);
akent(bigloc, 0, 180, -90, 90, [90 0], 1, '(d) Right');
```

```
% file akent.m
% analyze localization data using Kent distribution
function akent(bigloc, az_l, az_h, el_l, el_h, vcoord, dosp, s);

frontdat = sel_ld(bigloc,az_l,az_h,el_l,el_h);
sloc = sort_ld(frontdat);
if (dosp)
        draw_sp(1);
end
view(hp2cart(vcoord));
[dat, sloc] = next_ld(sloc);
while (length(dat) > 0)
        plot_sp(dat(1,[1 2]),'b+',6);            % plot target location
        [G,kappa,beta,q,ellz,ell,ln,isk] = kent_sp(dat(:,[3 4]));
        if (isk)
                ecol = 'w-';
        else
                ecol = 'w:';
        end
    plot_sp(ln(1,:),'w.',8);
    line_sp([dat(1,[1,2]); ln(1,:)],5,'w-');
    line_sp(ell,2,ecol)
    [dat, sloc] = next_ld(sloc);
end
h=text(1,-1.25,1,s);
set(h, 'FontSize', 10);
set(h, 'Color', 'b');
```

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