Reconfigurable Computing

Case Study: Kernel Adaptive Filters

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Overview

- Motivation
- Kernel methods
  - Vector processor
  - Pipelined
  - Braided
  - Distributed
- Conclusion

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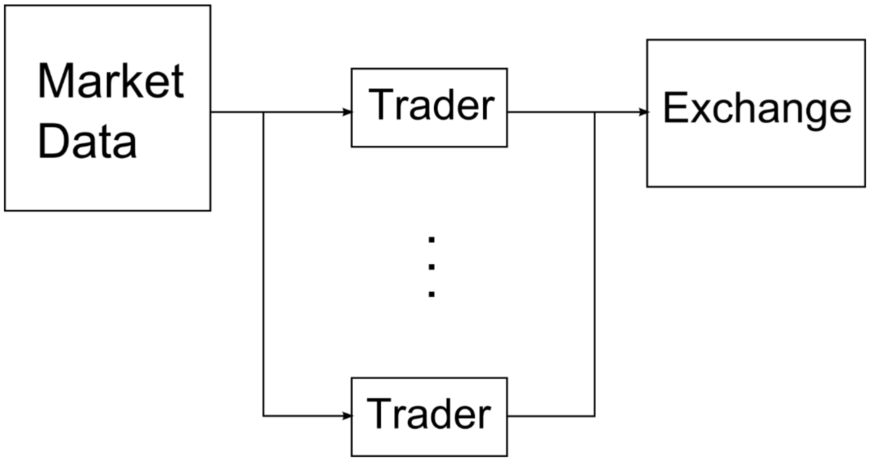
Motivation (latency)

How to beat other people to the money (latency)

› Low latency trading looks to trade in transient situations where market equilibrium disturbed

- 1ms reduction in latency can translate to $100M per year



› Latency also important: prevent blackouts due to cascading faults, turn off machine before it damages itself, etc

Information Week: Wall Street's Quest To Process Data At The Speed Of Light

Motivation (latency)

Exablaze Low-Latency Products

ExaLINK Fusion 48 SFP+ port layer 2 switch for replicating data typical 5 ns fanout, 95 ns aggregation, 110 ns layer 2 switch

Xilinx Ultrascale KU115 FPGA, QDR SRAM, ARM processor

ExaNIC X10 typical raw frame latency 60 bytes 780 ns

What we can't do: ML with this type of latency

Source: exablaze.com

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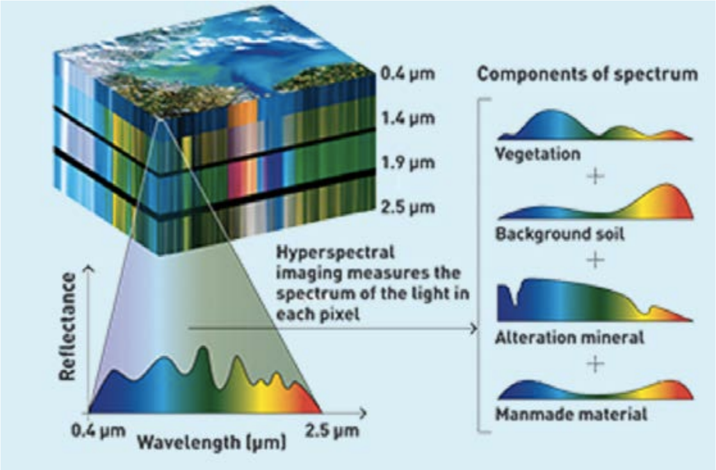
Motivation (throughput)

› Ability to acquire data improving (networks, storage, ADCs, sensors, computers)

- e.g. hyperspectral satellite images, Big Data e.g. SIRCA has 3PB of historical trade data

› Significant improvements in ML algorithms

- Deep learning (model high-level abstractions in data) for leading image and voice recognition problems; support vector machines to avoid overfitting



What we can't do: learning with this data rate

EPIC Technology Needed

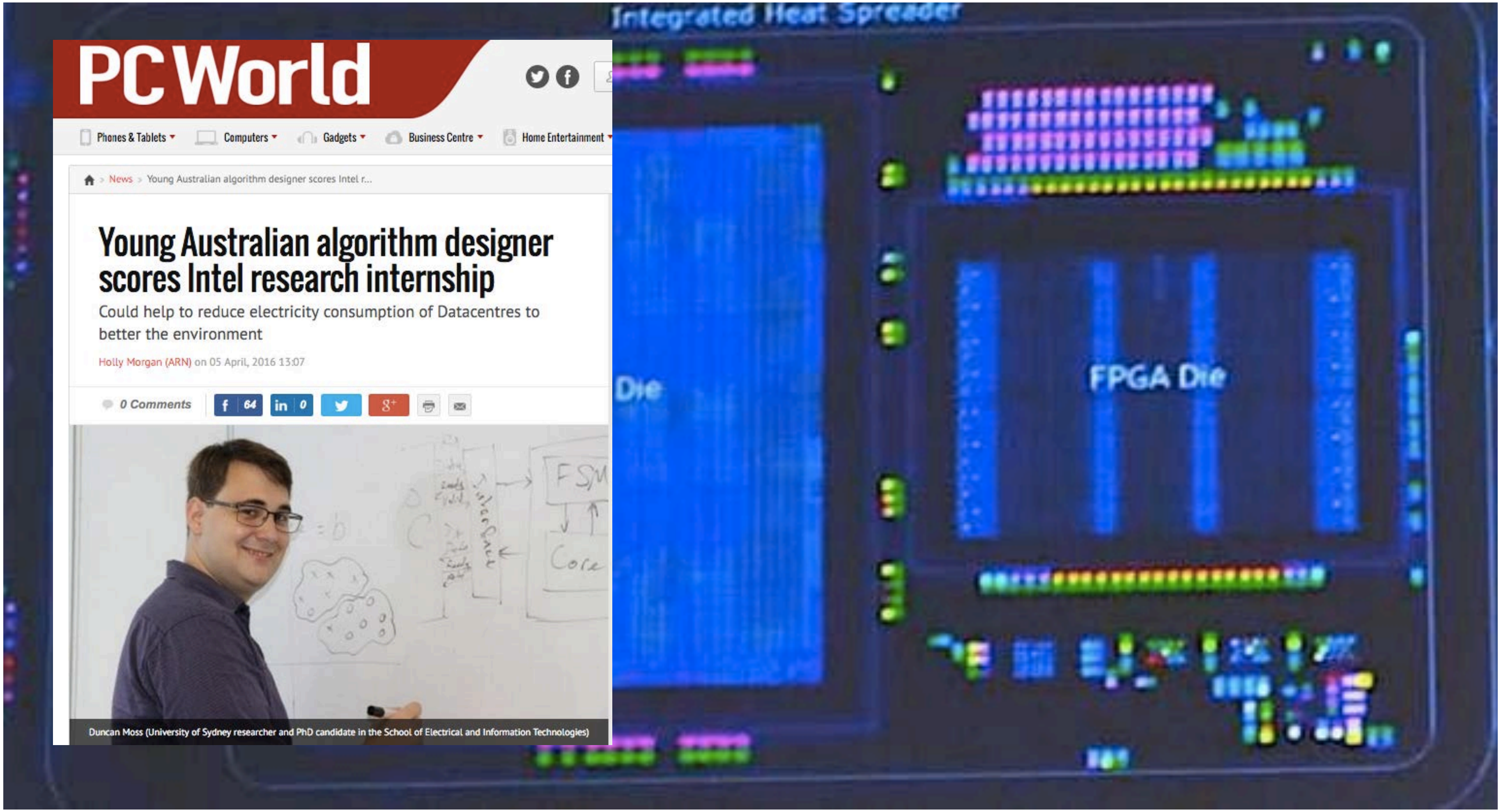
› To provide ML algorithms with higher throughput and lower latency we need

- Low **E**nergy – so power doesn’t become a constraint, operate off batteries (satellite and mobile)
- **P**arallelism – so we can reduce latency and increase throughput
- **I**nterface – so we don’t need to go off-chip which reduces speed and increases energy
- **C**ustomisable – so we can tailor entire design to get best efficiency

› **Using FPGAs, develop improved algorithms and system implementations for ML**

Why is FPGA research timely?

(Intel paid $16.7B for Altera; Intel presentation at OCP Summit 14/3/2016)



Overview

› Motivation

› **Kernel methods**

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› Conclusion

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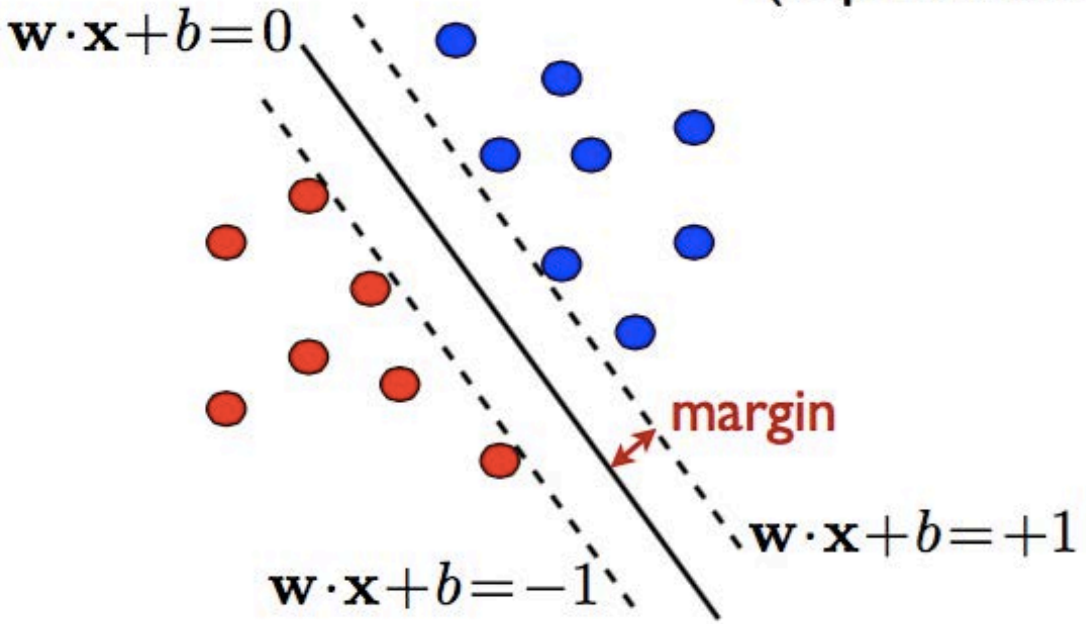
Linear Techniques

- Linear techniques extensively studied
  - Solution has form y = $\mathbf{w}^T\mathbf{x}$ + b
  - Use training data x to get maximum likelihood estimate of **w** or a posterior distribution of **w**
- Pros
  - Sound theoretical basis
  - Computationally efficient
- Cons
  - Linear!
- There is an equivalent dual representation

$$f(x) = \langle w, x \rangle + b = \sum \alpha_i y_i \langle x_i, x \rangle + b$$

e.g. Max Margin Hyperplane

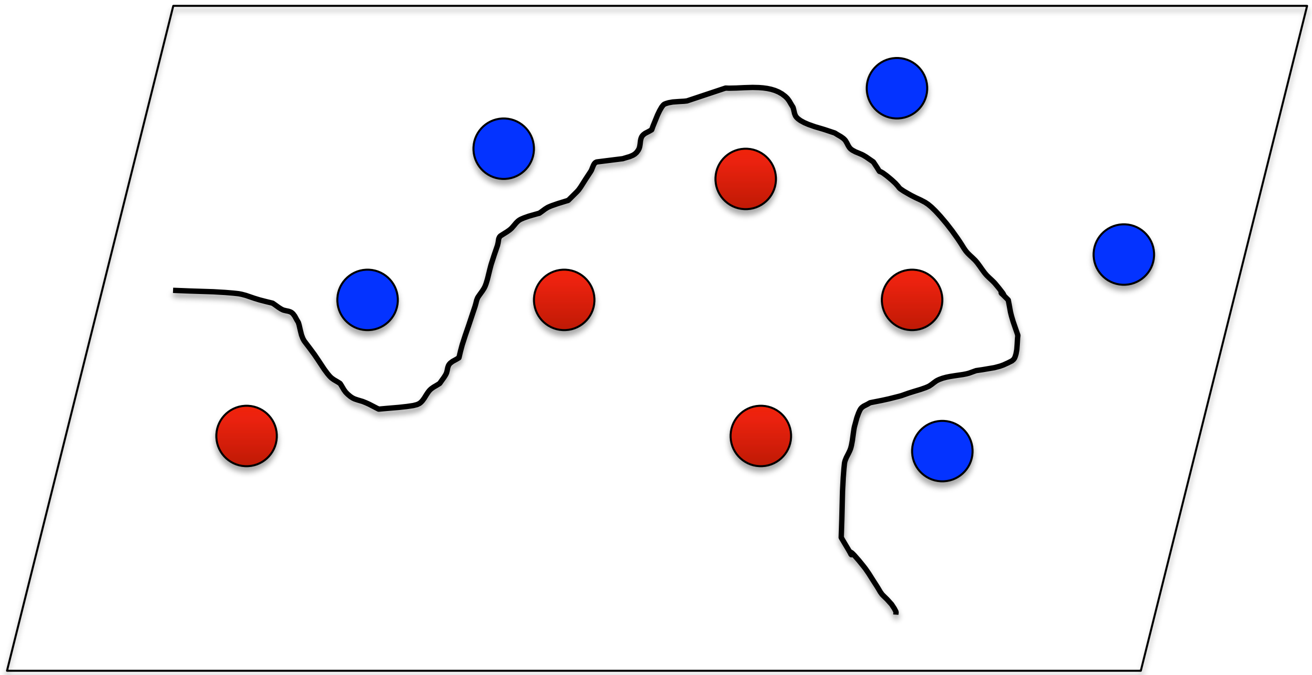
(Vapnik and Chervonenkis, 1964)



$$\min_{\mathbf{w},b} \quad \frac{1}{2}\|\mathbf{w}\|^2$$

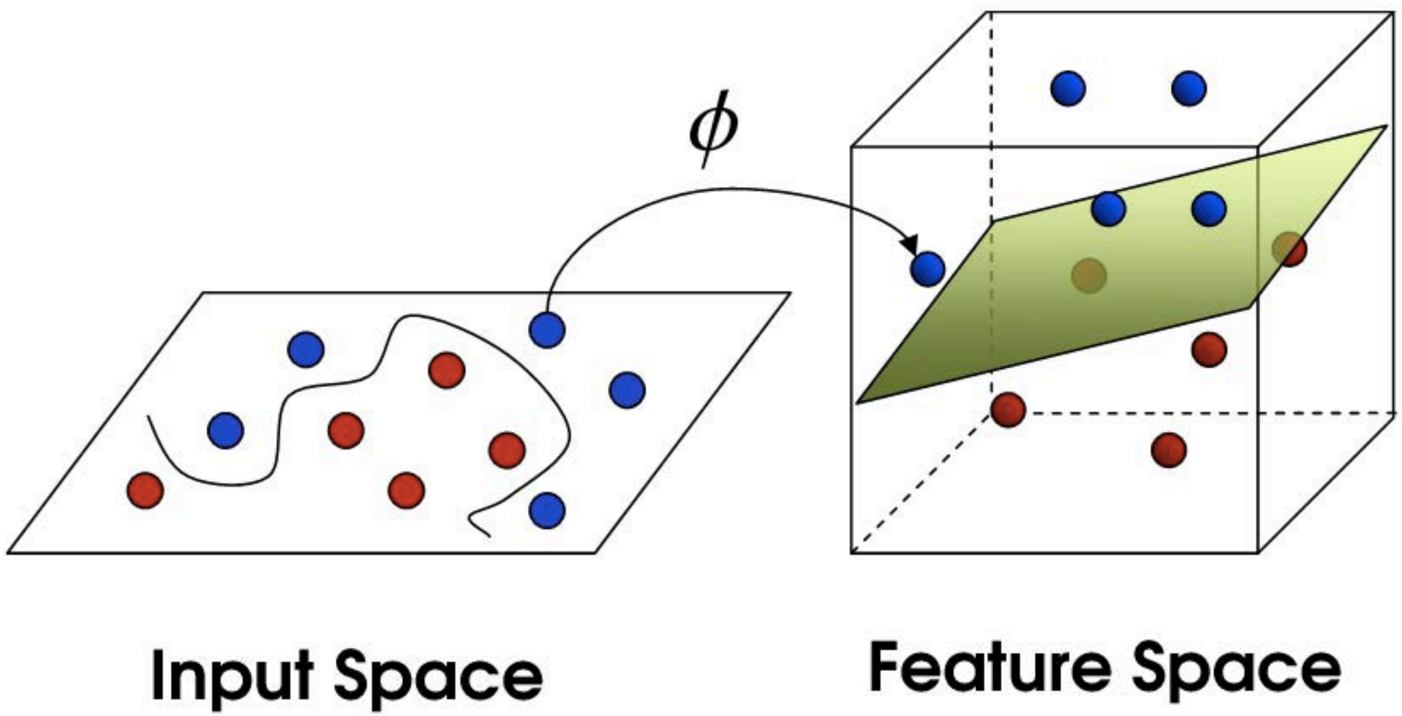
$$\text{subject to} \quad y_i(\mathbf{w}\cdot\mathbf{x}_i + b) \geq 1, i \in [1, m].$$

What do we do if given this problem?



› Map the problem to a **feature space**

Mapping to a Feature Space



- Choose high dimensional feature space (so easily separable)
- **BUT computing Φ is expensive!**

Kernel Trick

› Kernel is a similarity function

- defined by an implicit mapping ϕ, (original space to feature space)

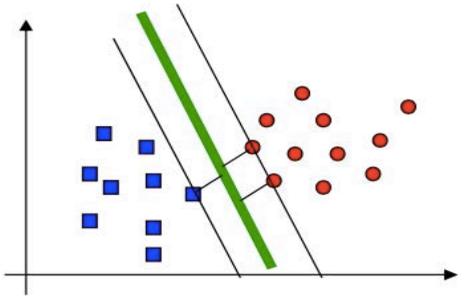
$$\kappa(x, x') = \phi(x)^T \phi(x') = \langle \phi(x), \phi(x') \rangle$$

- e.g. Linear kernel $\kappa(x,x') = \langle x, x' \rangle$
- e.g. Polynomial kernel $\kappa(x,x') = (1 + \langle x, x' \rangle)^d$ for d=2: $\phi(x) = (x_1^2, x_2^2, \sqrt{2}x_1x_2)$
- e.g. Gaussian kernel (universal approximator) $k(x, x') = \exp\left(-\frac{\|x - x'\|^2}{2\,\sigma^2}\right)$
  - $\Phi(x)$ infinite in dimension!

› **Modify linear ML techniques to kernel ones by replacing dot products with the kernel function (kernel trick)**

- e.g. linear discriminant analysis, logistic regression, perceptron, SOM, K-means, PCA, ICA, LMS, RLS, …
- **While we only describe prediction here, also applied to training equations**

Support Vector Machine



- **The decision boundary:**

$$\hat{\mathbf{w}}^T\mathbf{x} + w_0 = \sum_{i \in SV} \hat{\alpha}_i y_i (\mathbf{x}_i{}^T \mathbf{x}) + \text{b}$$

- **Classification decision:**

$$\hat{y} = \text{sign}\left[\sum_{i \in SV} \hat{\alpha}_i y_i (\mathbf{x}_i{}^T \mathbf{x}) + \text{b}\right]$$

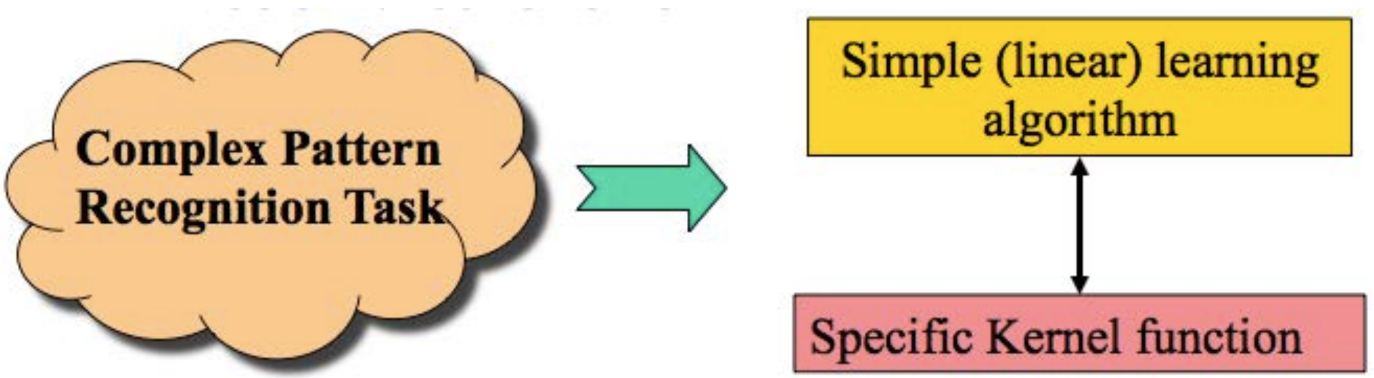
$$\mathbf{x} \rightarrow \varphi(\mathbf{x})$$

$$K(\mathbf{x}, \mathbf{x}') = \varphi(\mathbf{x})^T \varphi(\mathbf{x}')$$

Never explicitly compute Φ(x), computing K(x,x') is O(m)
e.g. poly kernel Φ(x), dimension (d+m-1)!/d!(m-1)!
For d=6, m=100 this is a vector of length 1.6e9

Approach

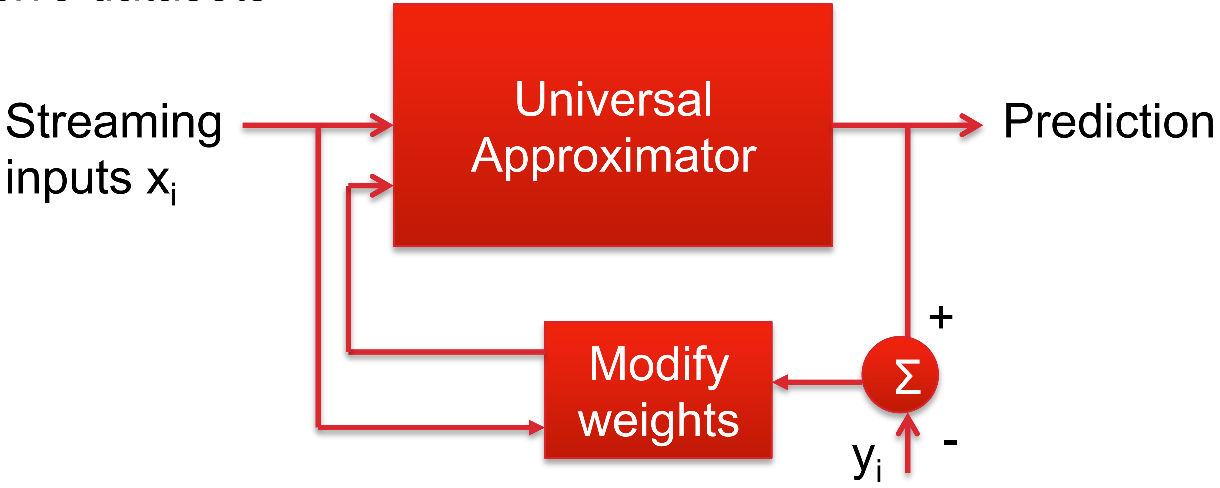
- In Kernel-based learning algorithms, problem solving is now decoupled into:
  - A general purpose learning algorithm often linear (well-founded, robustness, …)
  - A problem specific kernel (we focus on time series but kernels exist for text, DNA sequences, NLP)



Online Kernel Methods

Examples are KLMS and KRLS

- Traditional ML algorithms are batch based
  - Make several passes through data
  - Requires storage of the input data
  - Not all data may be available initially
  - Not suitable for massive datasets
- Our approach: online algorithms
  - Incremental, inexpensive state update based on new data
  - Single pass through the data
  - Can be high throughput, low latency



Kernel Online Algorithms

Two extensively studied types of online kernel methods:

› Kernel Least Mean Squares (KLMS)
  - $O(N)$
  - Converges slowly (steepest descent)
  - Takes a 'step' towards minimising the instantaneous error
  - e.g. KNLMS, NORMA

› Kernel recursive least squares (KRLS)
  - $O(N^2)$
  - Converges quickly (Newton Raphson)
  - Directly calculates least squares solution based on previous training examples using Matrix Inversion Lemma (matrix-vector multiplication)
  - e.g. SW-KRLS

Convergence

RICHARD *et al.*: ONLINE PREDICTION OF TIME SERIES DATA WITH KERNELS

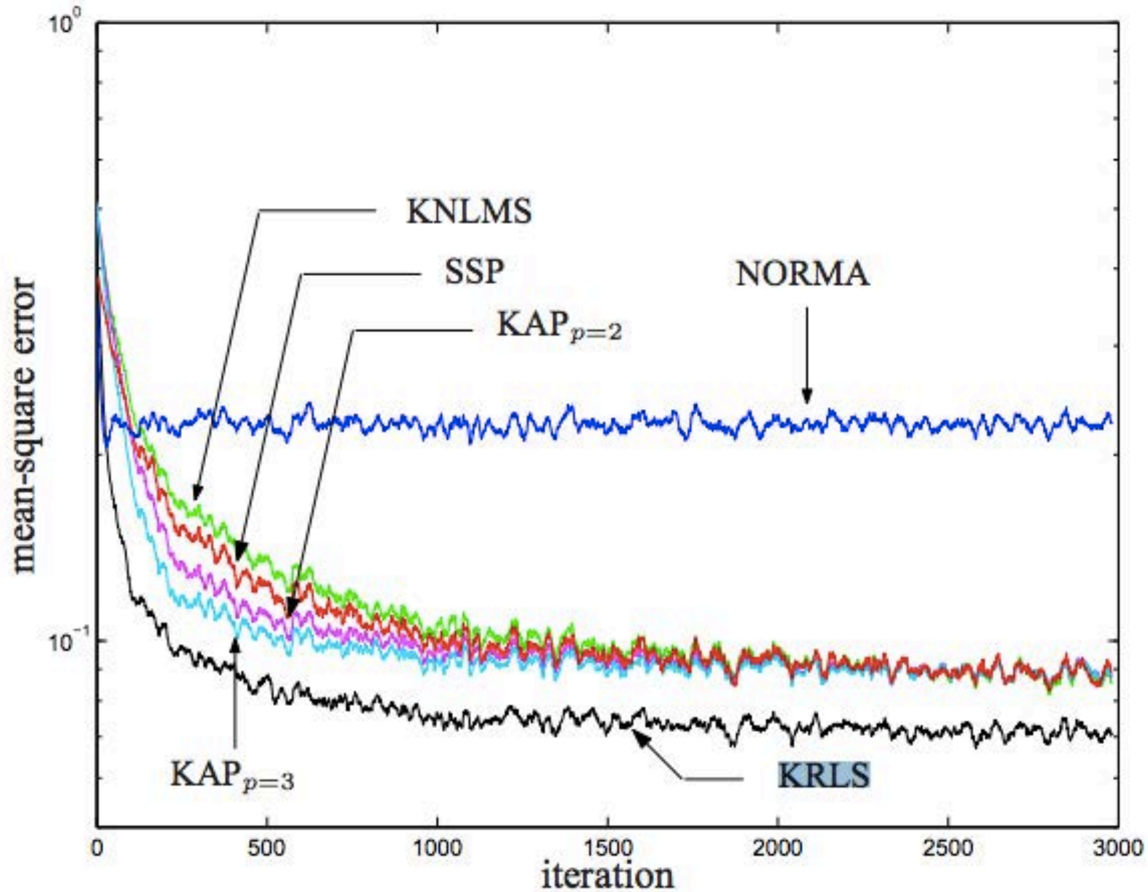


Fig. 2. Learning curves for KAP, KNLMS, SSP, NORMA and KRLS obtained by averaging over 200 experiments.

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SW-KRLS Algorithm

The pseudo code of the SW-KRLS algorithm

Initialize $K_0 = (1+c)I$ and $K_0^{-1} = I/(1+c)$.

for n=1,2.. **do**

  Get $\tilde{K}_n$ from K_{n-1} with Eq.(1)

  Calculate $\tilde{K}_{n-1}^{-1}$ with Eq.(2)

  Get K_n with Eq.(3)

  Calculate K_n^{-1} with Eq.(4)

  Get the updated solution $\alpha_n = K_n^{-1} Y_n$

end for

Computation complexity: $O(N^2)$.

$$\tilde{K}_n = \begin{bmatrix} K_{n-1} & k_n(x_n) \\ k_n(x_n)^T & k_{nn} + c \end{bmatrix} \quad (1)$$

$$\hat{\mathbf{K}}_n^{-1} = \begin{bmatrix} \mathbf{K}_{n-1}^{-1}(\mathbf{I} + \mathbf{b}\mathbf{b}^T \mathbf{K}_{n-1}^{-1T} g) & -\mathbf{K}_{n-1}^{-1}\mathbf{b}g \\ -(\mathbf{K}_{n-1}^{-1}\mathbf{b})^T g & g \end{bmatrix} \quad (2)$$

where $b = k_{n-1}(x_n)$ $d = k_{nn} + c$ $g = (d - b^T K_{n-1}^{-1} b)^{-1}$

$$K_n = \begin{bmatrix} k_{n-N,n-N} + c & p^T \\ p & \tilde{K}_{n-1} \end{bmatrix} \quad (3)$$

where $p = [k(x_{n-N}, x_{n-N+1}), \ldots, k(x_{n-N}, x_{n-1})]^T$

$$K_n^{-1} = G - \frac{ff^T}{e} \quad (4)$$

where $\tilde{K}_n^{-1} = \begin{bmatrix} e & f^T \\ f & G \end{bmatrix}$

Key idea

Vector add C = A + B

- **Microprocessor** O(N) cycles

```
for (i = 0; i < N; i++)
    C[i] = A[i] + B[i];
```

- **Vector processor** O(1) cycle

```
VADD(C, A, B)
```

- Implemented as a custom KRLS vector processor using FPGA technology

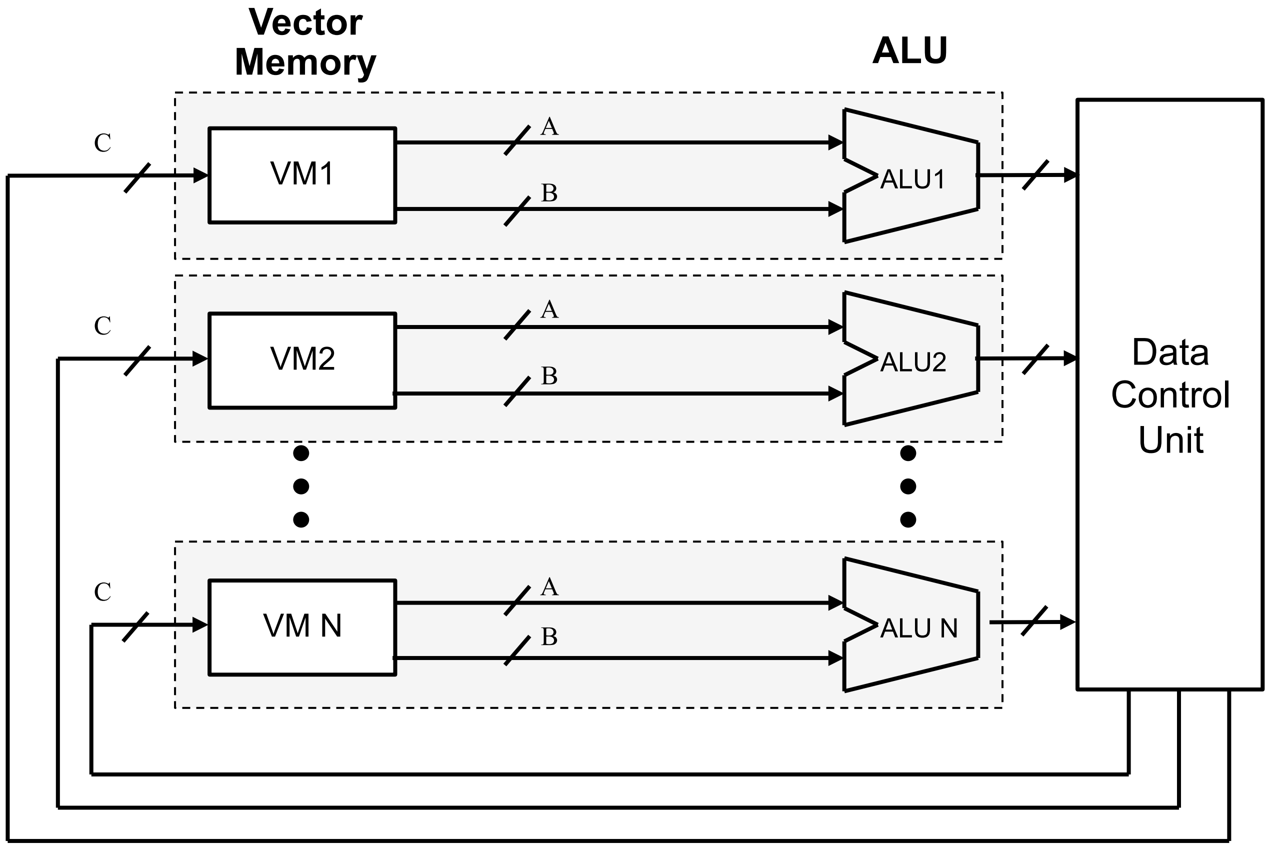
Instruction Set

ALL i , j AND L INDEXES RANGE FROM 1 TO N

Microcode (Opcode)	Function	Total Cycles
NOP(000)	No operation	1
BRANCH (0111)	BRANCH	4
VADD (0001)	**Vector add**	**14**
VSUB (0010)	**Vector subtract**	**14**
VMUL (0011)	**Array multiply**	**10**
VDIV (0100)	**Vector divide**	**N+28**
VEXP (0110)	**Vector exponentiation**	**N+21**
S2VE (1000)	**Clone a vector N times**	**N+4**
PVADD (1001)	***N x Vector add***	***N+13***
PVSUB (1010)	***N x Vector subtract***	***N+13***
PVMUL (1011)	***N x Vector multiply***	***N+9***
PVDOT (0101)	***N x Vector dot product***	***N+9+10***

SW-KRLS and other kernel methods implemented efficiently using this simple instruction set

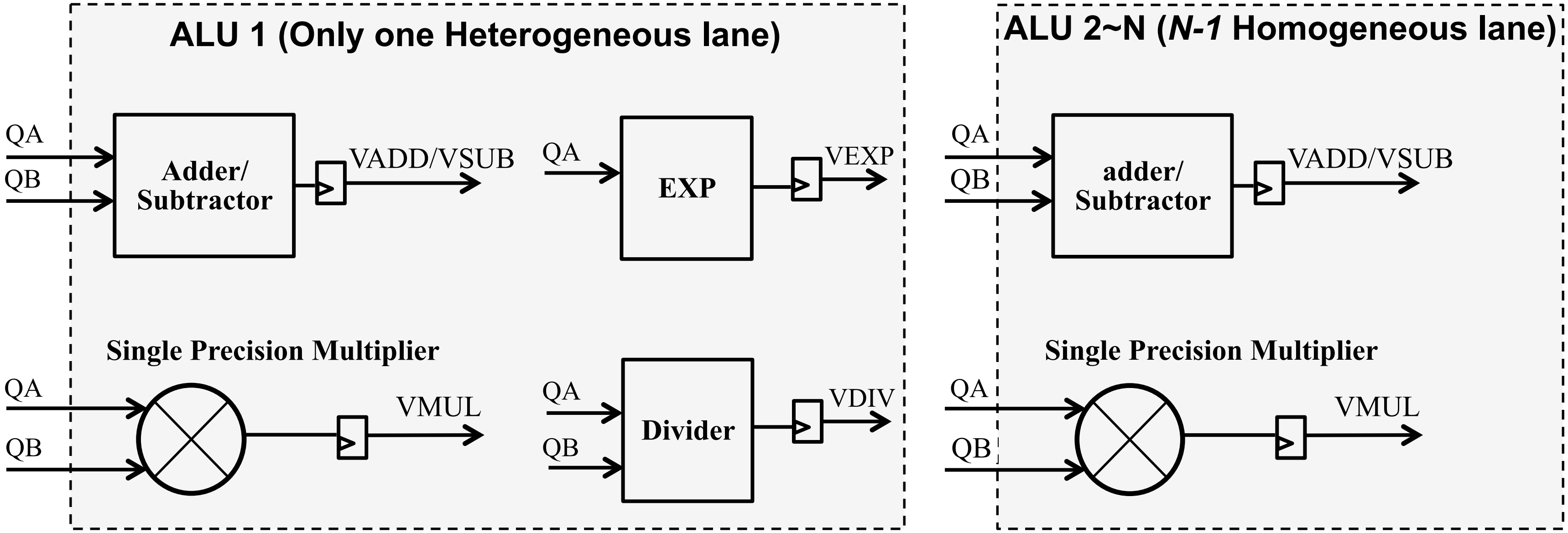
Datapath



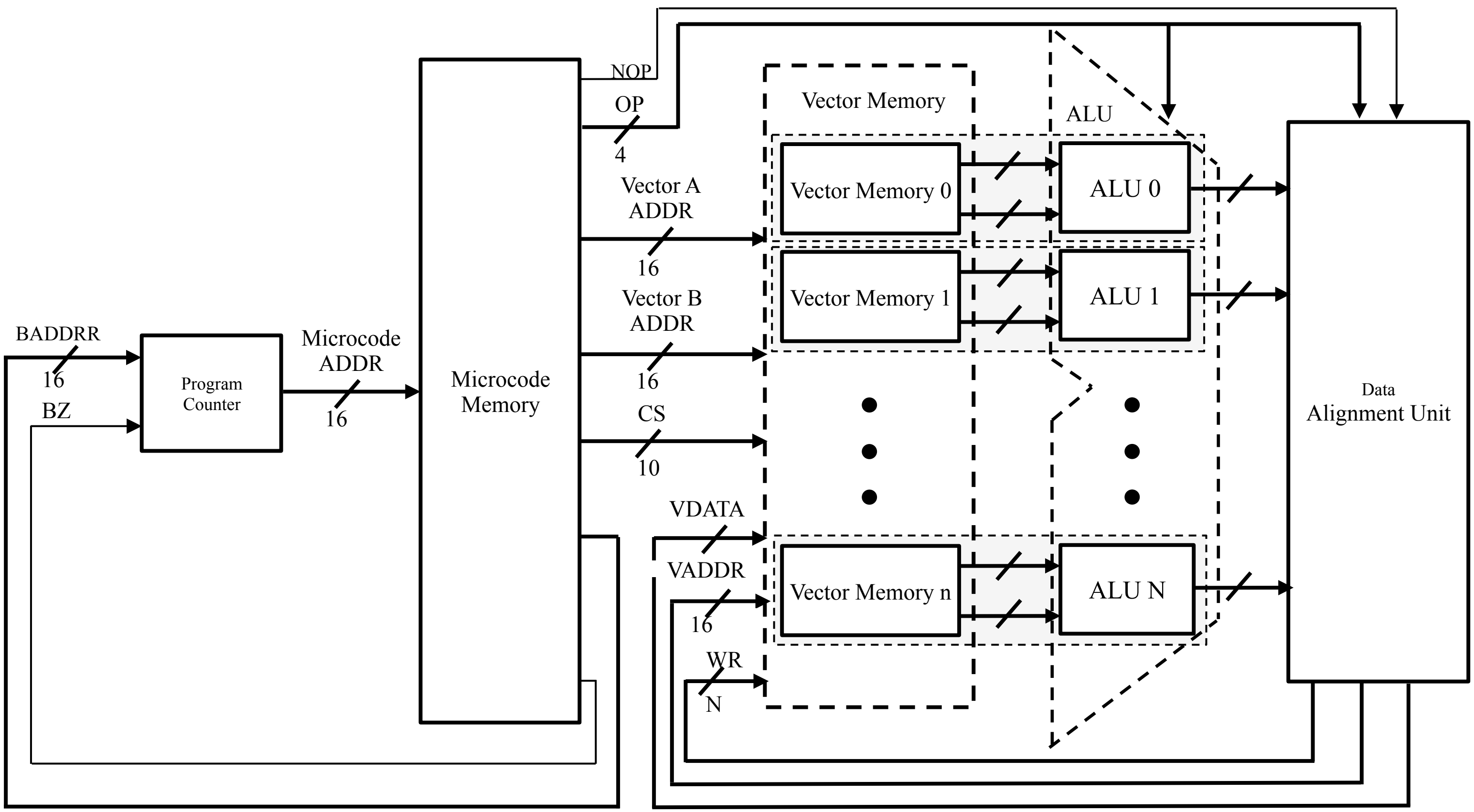
Vector Memory
ALU
C
VM1
A
B
ALU1
C
VM2
A
B
ALU2
C
VM N
A
B
ALU N
Data Control Unit

ALU architecture

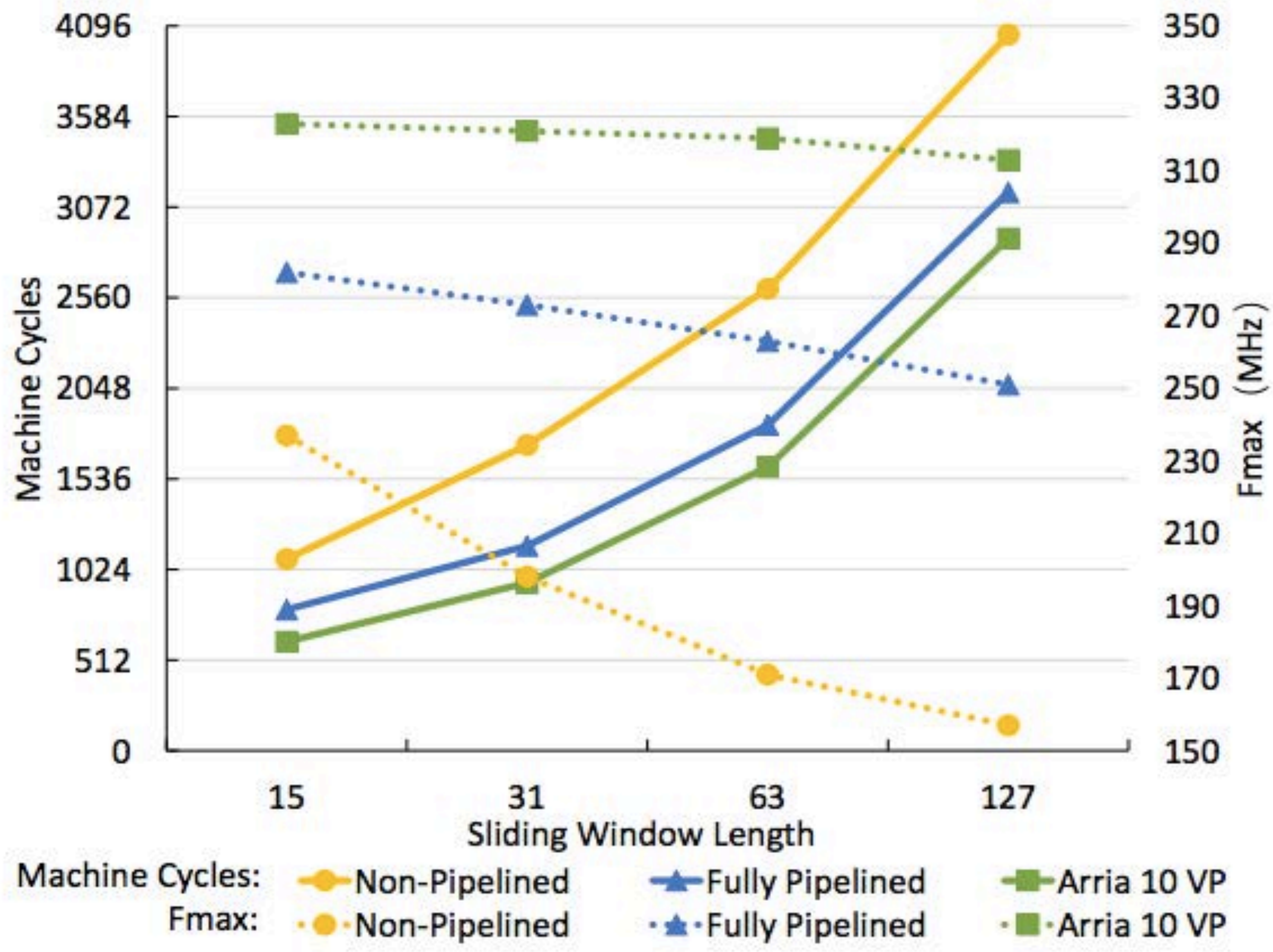
- ALU 1 - adder, multiplier, exp and divider
- ALU 2 .. ALU N - only adder and multiplier



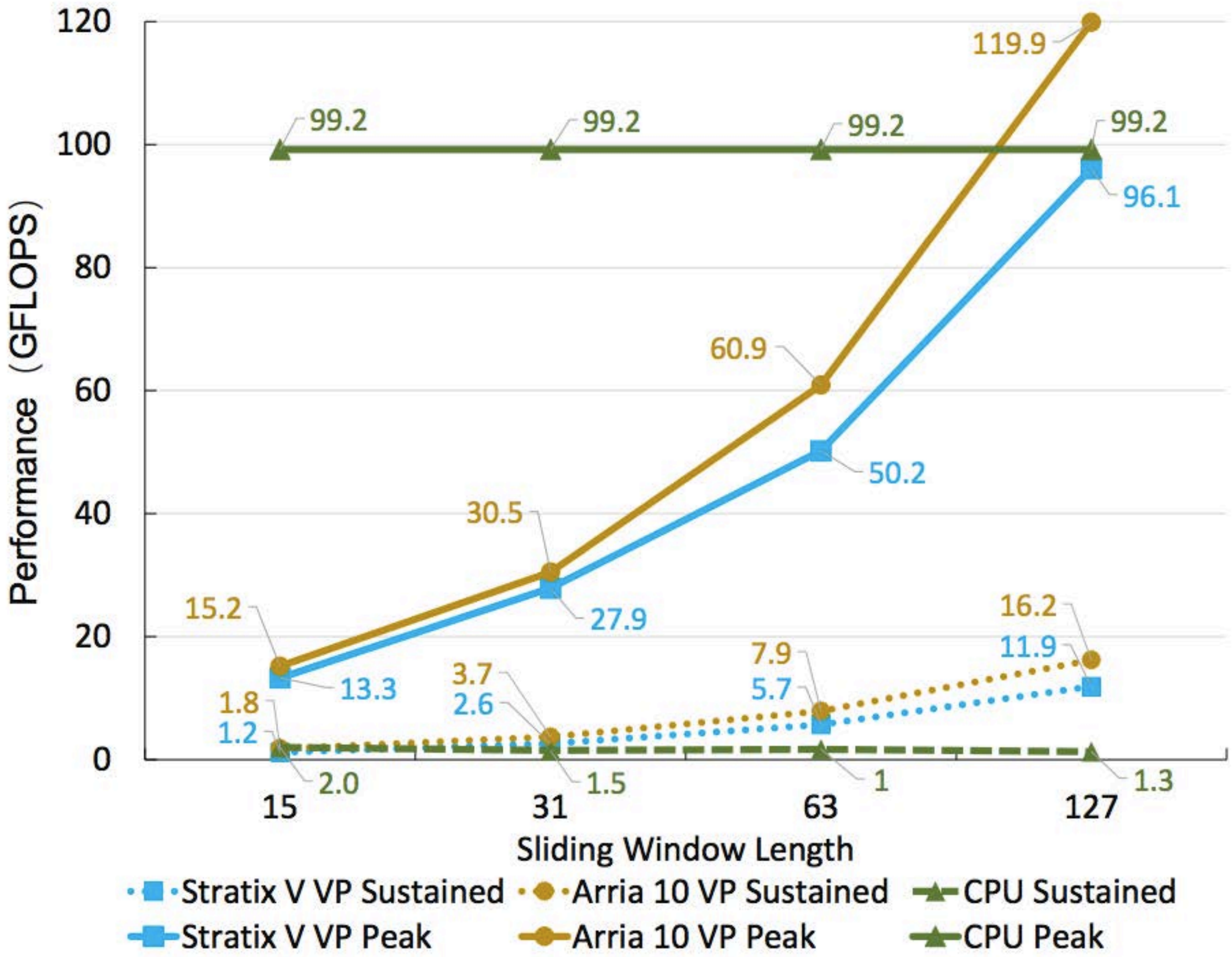
Detailed Datapath



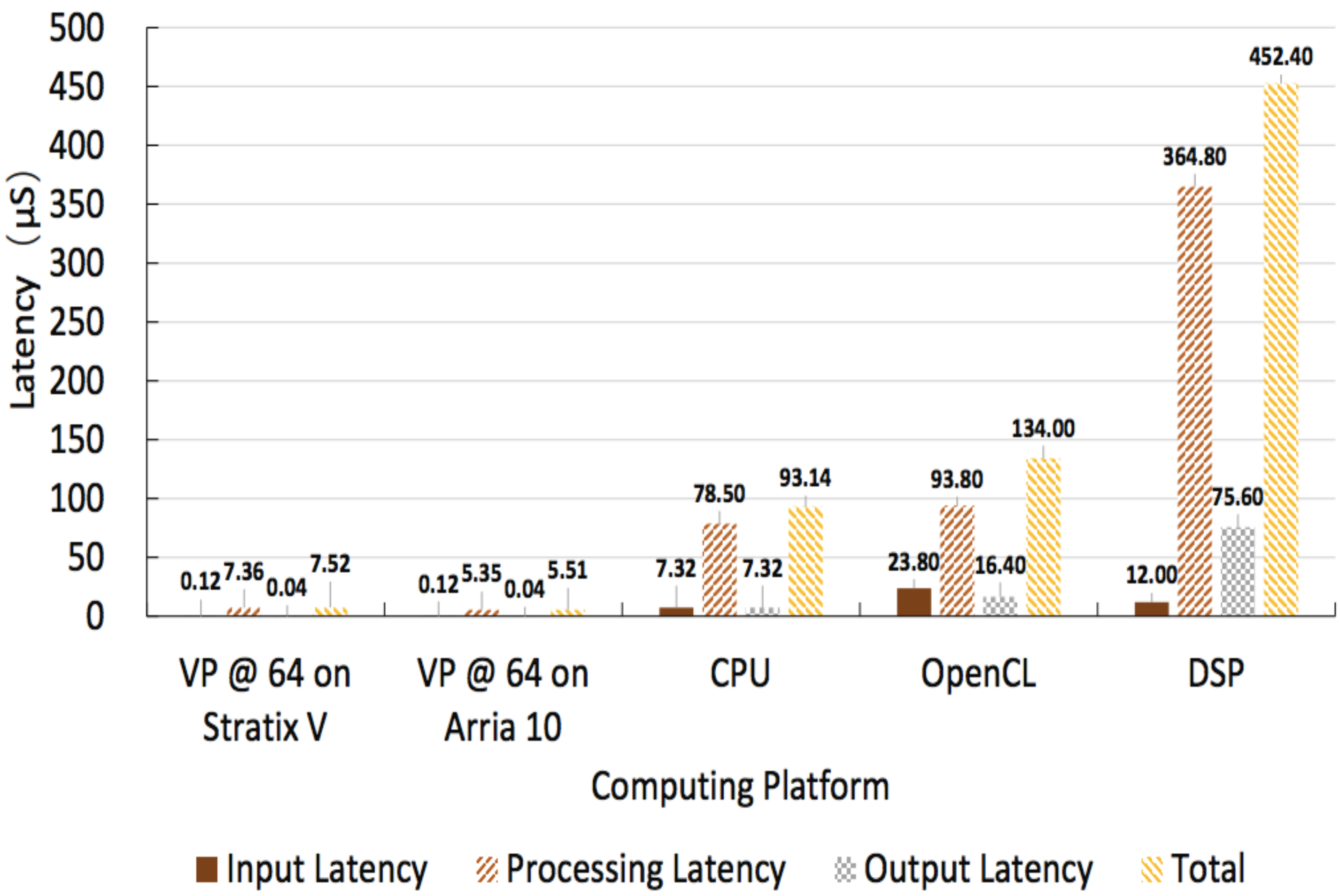
Arria 10 vs Stratix V vs CPU Single P&R



Arria 10 vs Stratix V vs CPU Single Performance



Arria 10 vs Stratix V vs CPU Single Latency



Performance Summary (Stratix V)

- SW-KRLS N=64

Platform	Power (W)	Latency (uS)	Energy (10^-5 J)
Our processor (DE5 5SGXEA7N)	2 (27)	**1 (12.6)**	**1 (34)**
DSP (TMS320C6678**)**	**1 (13)**	355 (4476)	181 (6167)
CPU (i5-2400@3.1GHz)	1 (13)	16 (201)	8 (269)

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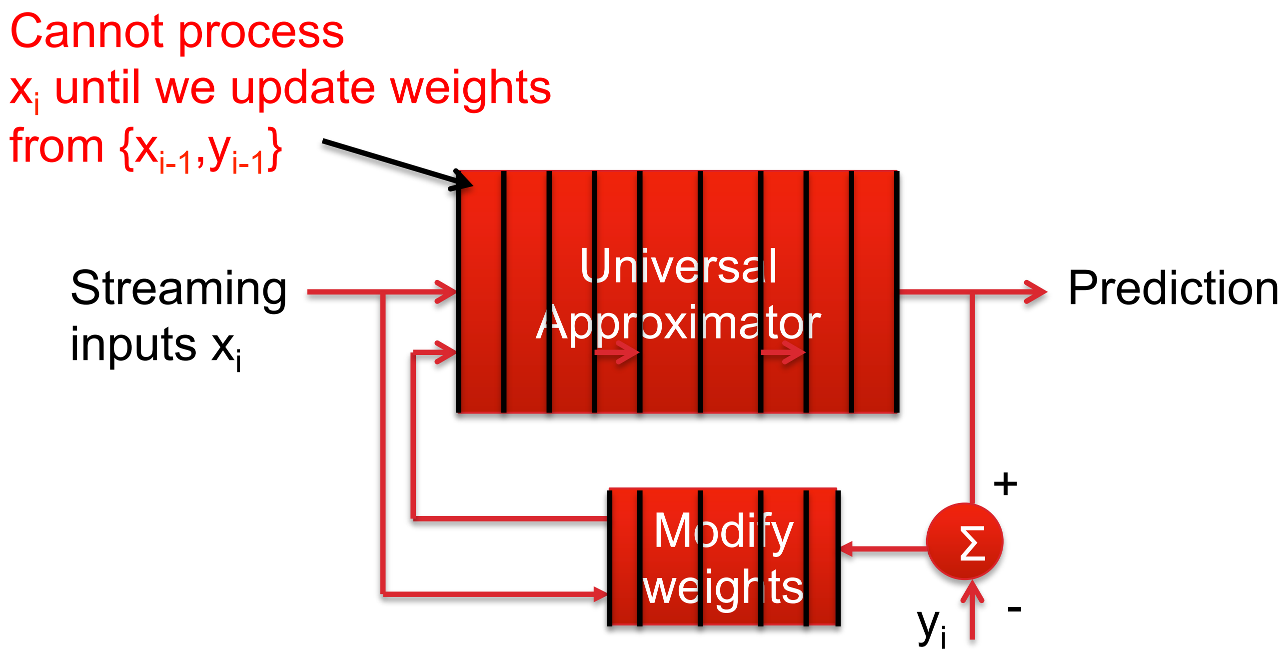
Obstacle to Pipelining

Dependency Problem



KNLMS Regression

- Finds D (dictionary which is subset of input vectors), and α (weights) for function

$$f(x) = \sum_{i=1}^{D} \alpha_i \kappa(x, d_i)$$

- Is a stochastic gradient descent style kernel regression algorithm. Given a new input/output pair, $\{x_n, y_n\}$, weight update is:
- 1. Evaluate κ between x_n and each entry of D_{n-1}, creating kernel vector, k.
- 2. If max(k) < μ_0, add x_n to the dictionary, producing D_n
- 3. Update the weights using:

$$\alpha_n = \alpha_{n-1} + \frac{\eta}{\varepsilon + k^T k}(y_n - k^T \alpha_{n-1})k$$

- How can we chose κ, μ_0, η and ϵ? We must do a parameter search.

Removing Dependencies

› Training is usually:

for (hyperparameters)

  for (inputs)

    learn_model()

› Alternative is to find L independent problems

- E.g. monitor L different things

› Our approach: run L independent problems (different parameters) in the pipeline

- Updates ready after L subproblems
- Less data transfer

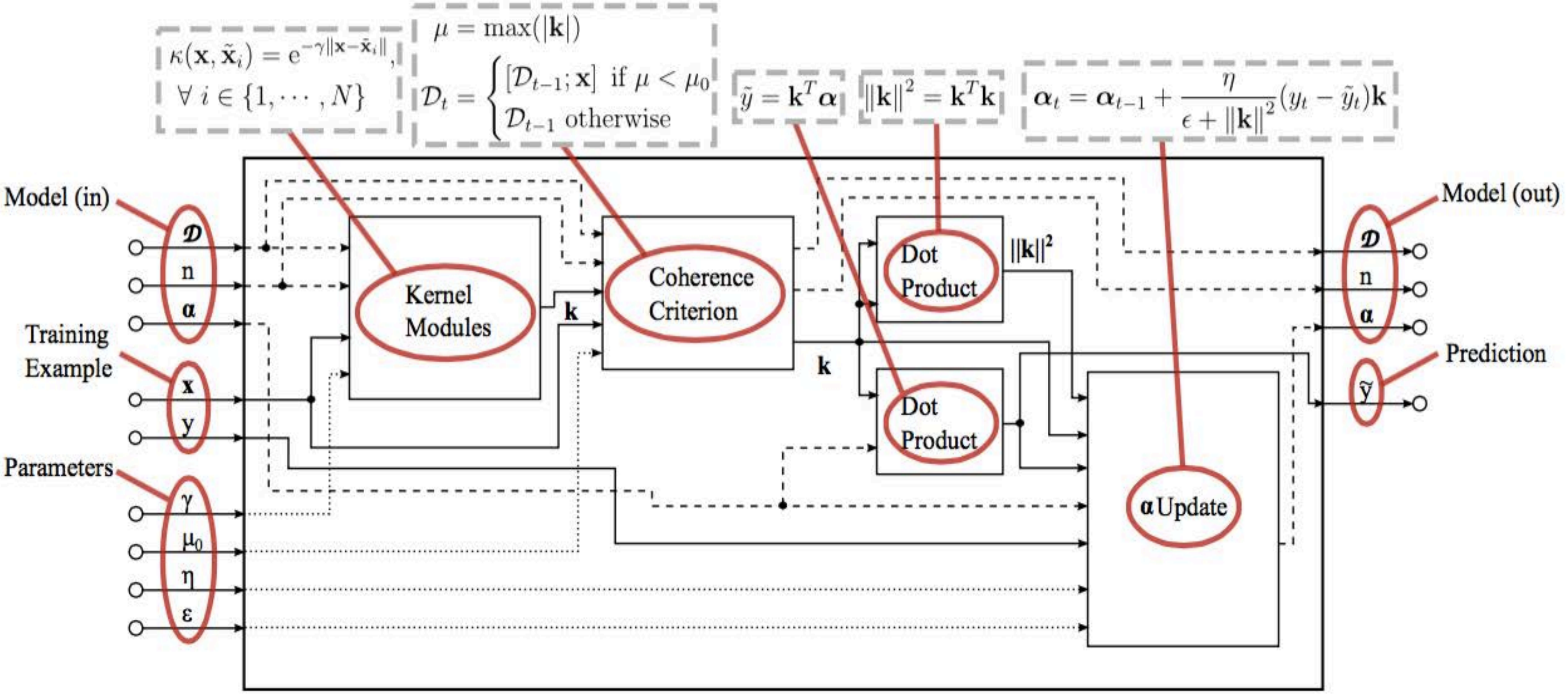
for (inputs)

  for (hyperparameters)

    learn_model()

- Similar approach for multiclass classification (train C(C-1)/2 binary classifiers)

High Throughput KNLMS



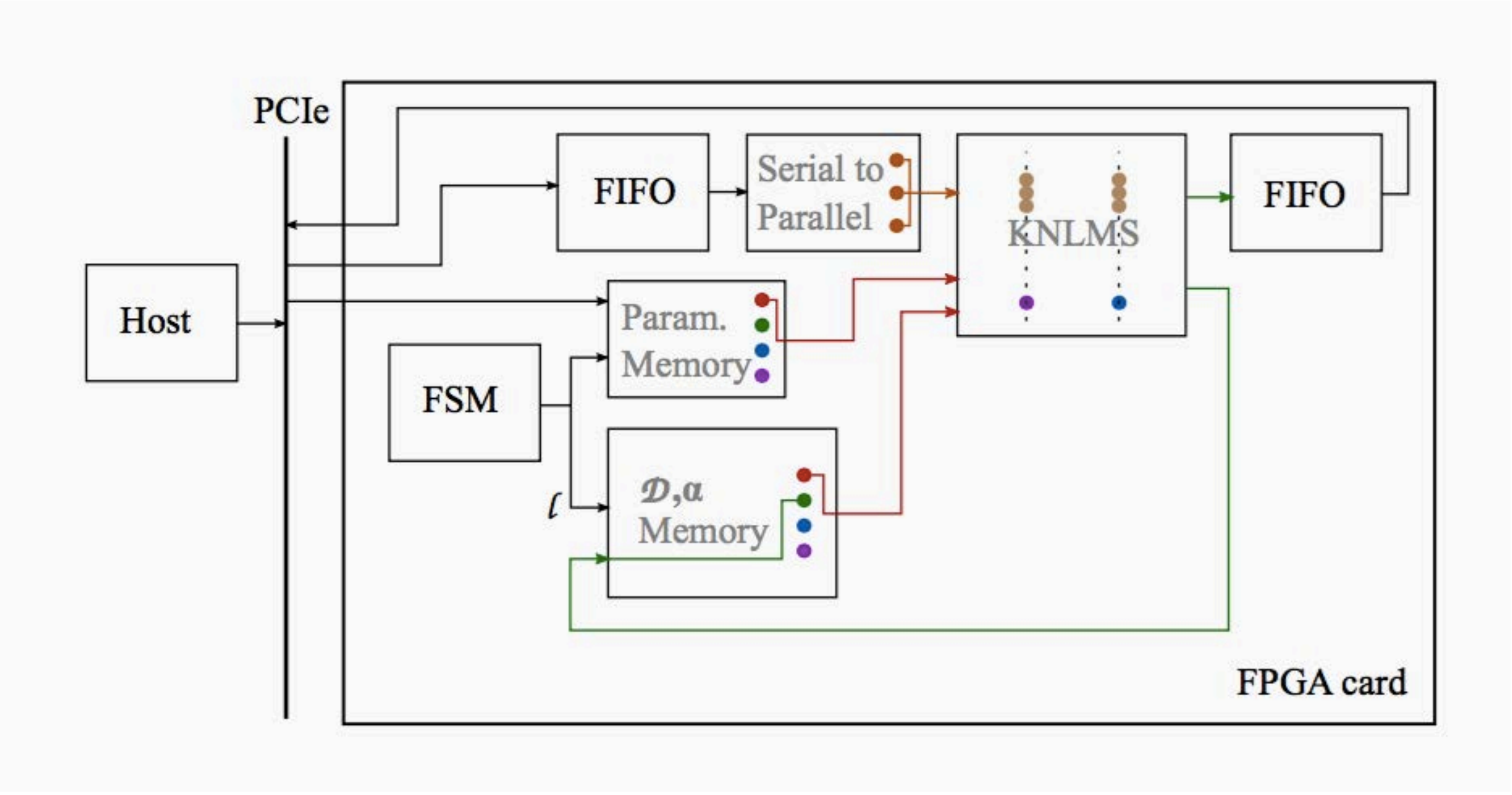
Scalability

- Area O(MN)
- Memory O(MN)
- Latency O($\log_2$N+$\log_2$M)

	+ (11)	× (7)	/ (30)	exp (20)	< (4)
Operation	$2MN + 2N$	$MN+$ $4N + 1$	1	N	$N - 1$
Latency	$\log_2 N+$ $\log_2 M + 3$	5	1	1	$\log_2 N$

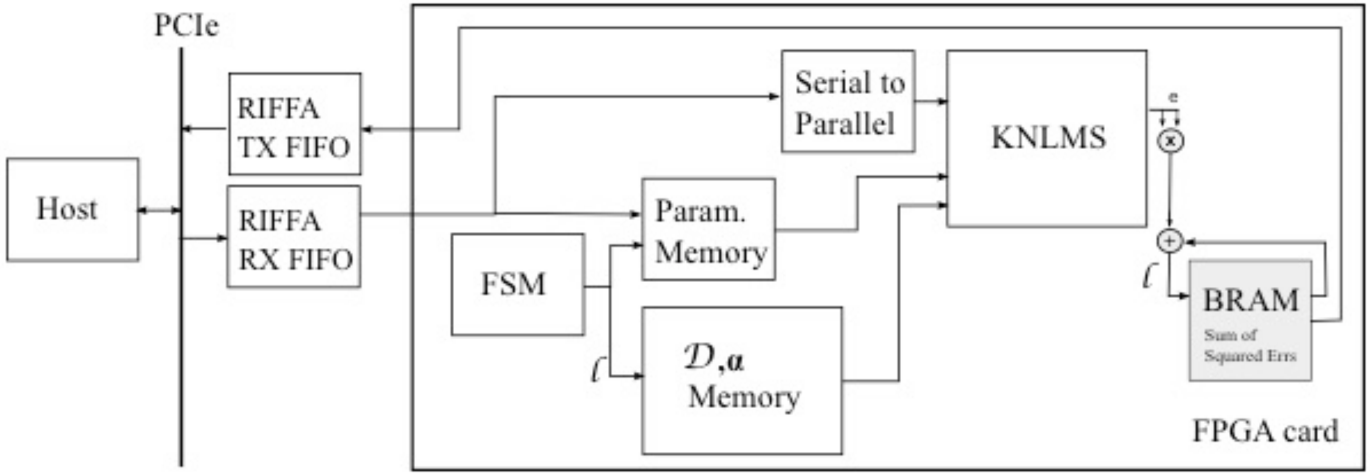
Original Implementation

- Break feedforward/feedback path and sythesised with Vivado HLS
- RIFFA 2.2.0 used for PCIe interface



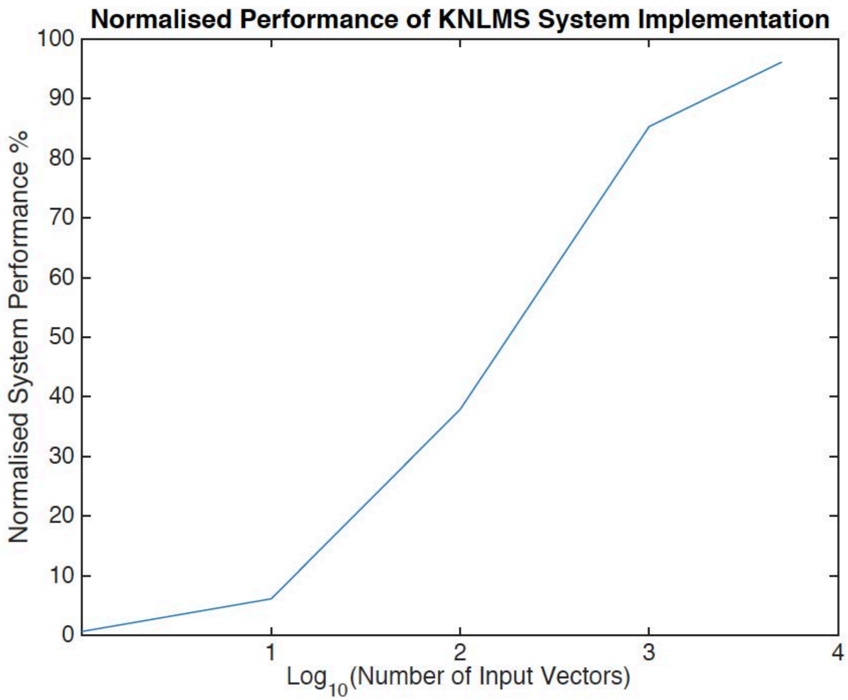
Optimised Search

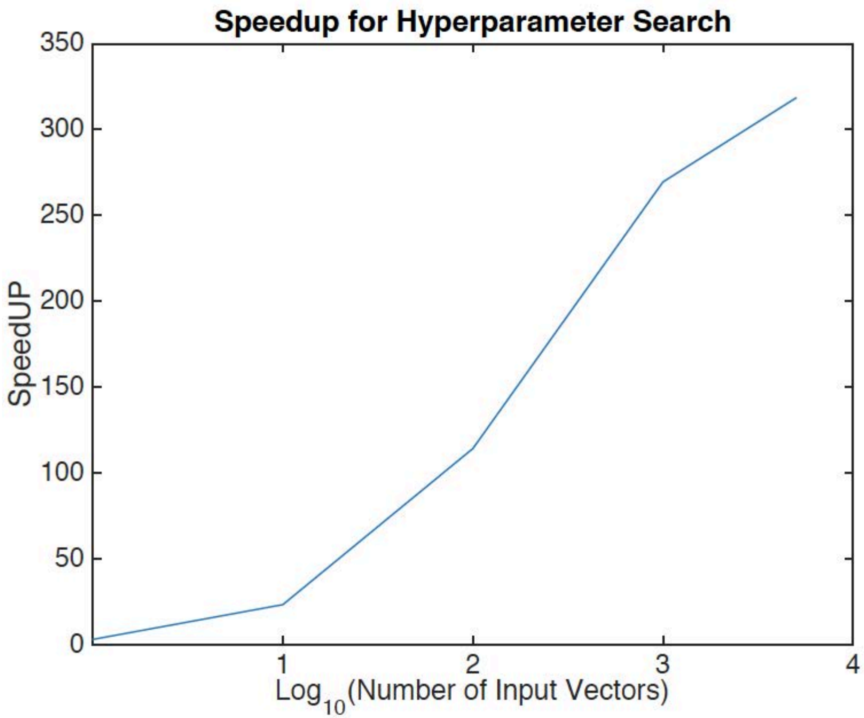
- Float KNLMS core integrated with RIFFA on Xilinx VC707 board (# ParamSets = 256)



- Original version read back all the predicted values
- Optimised by send all data and then read back accumulated square error
- 3x faster than original interface

Performance of KNLMS Core vs Number of Train Samples





- 96% of the peak core performance has been achieved with RIFFA integration when # of train samples reaches 4999

Performance

Core with input vector M=8 and dictionary size N=16 (KNLMS)

Implementation	Freq (MHz)	Time (ns)	Slowdown
Float	314	3	1
System	250	4	1.3
Naive	97	7,829	2,462
CPU (C)	3,600	940	296
Pang et al (2013)	282	1,699	566

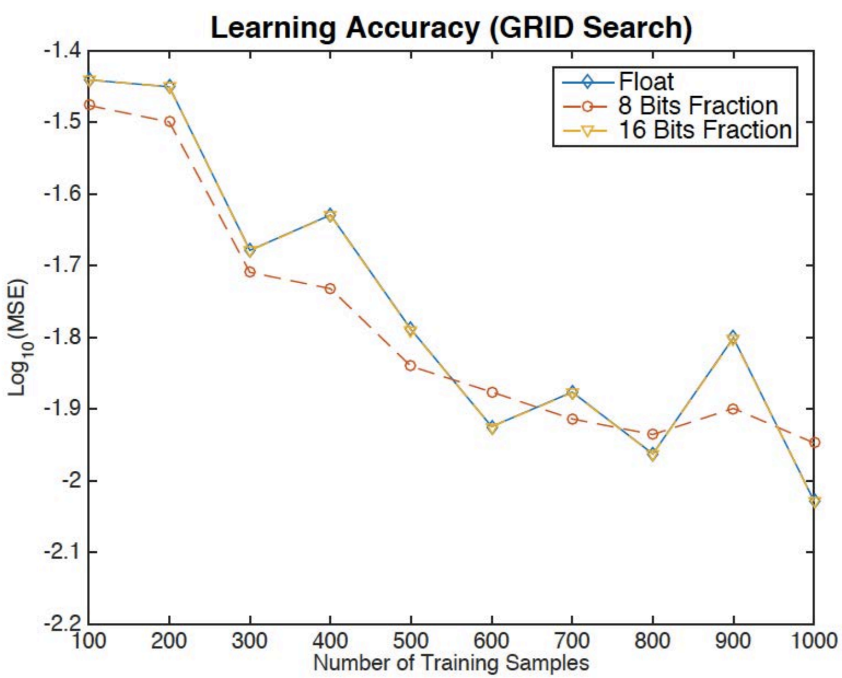
- Energy efficient, Parallelism (pipelining), Integrated with PCIe and Customised (problem changed to remove dependencies)
- Can do online learning from 200 independent data streams at 70 Gbps (160 GFLOPS)

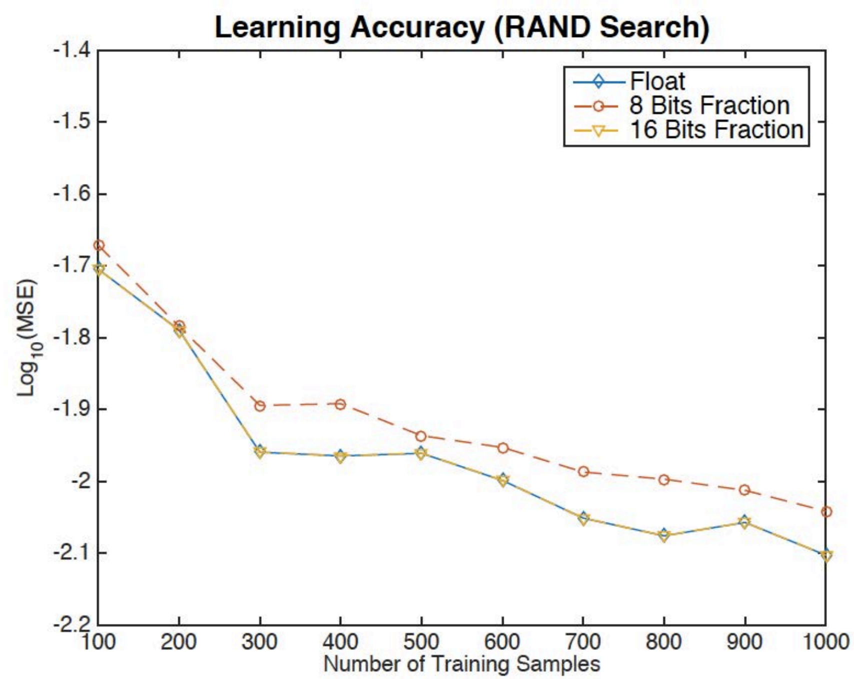
Precision and Learning Accuracy for KNLMS

- Trained and Tested with Mackey Glass data

- Trained with 100-1000 data samples/Tested with 999 data samples

- Hyperparameter set chosen from Grid/Random Search





- 16 fraction bits are enough to produce equivalent learning accuracies with Float

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NORMA

Naive Online regularised Risk Minimization Algorithm

› Finds D (dictionary which is subset of input vectors), and α (weights) for function

$$f(x) = \sum_{i=1}^{D} \alpha_i \kappa(x, d_i)$$

› Minimise instantaneous risk of predictive error ($R_{inst,\lambda}$) by taking a step in direction of gradient

$$f_{t+1} = f_t - \eta_t \partial_f R_{inst,\lambda}[f, x_{t+1}, y_{t+1}]\Big|_{f=f_t}$$

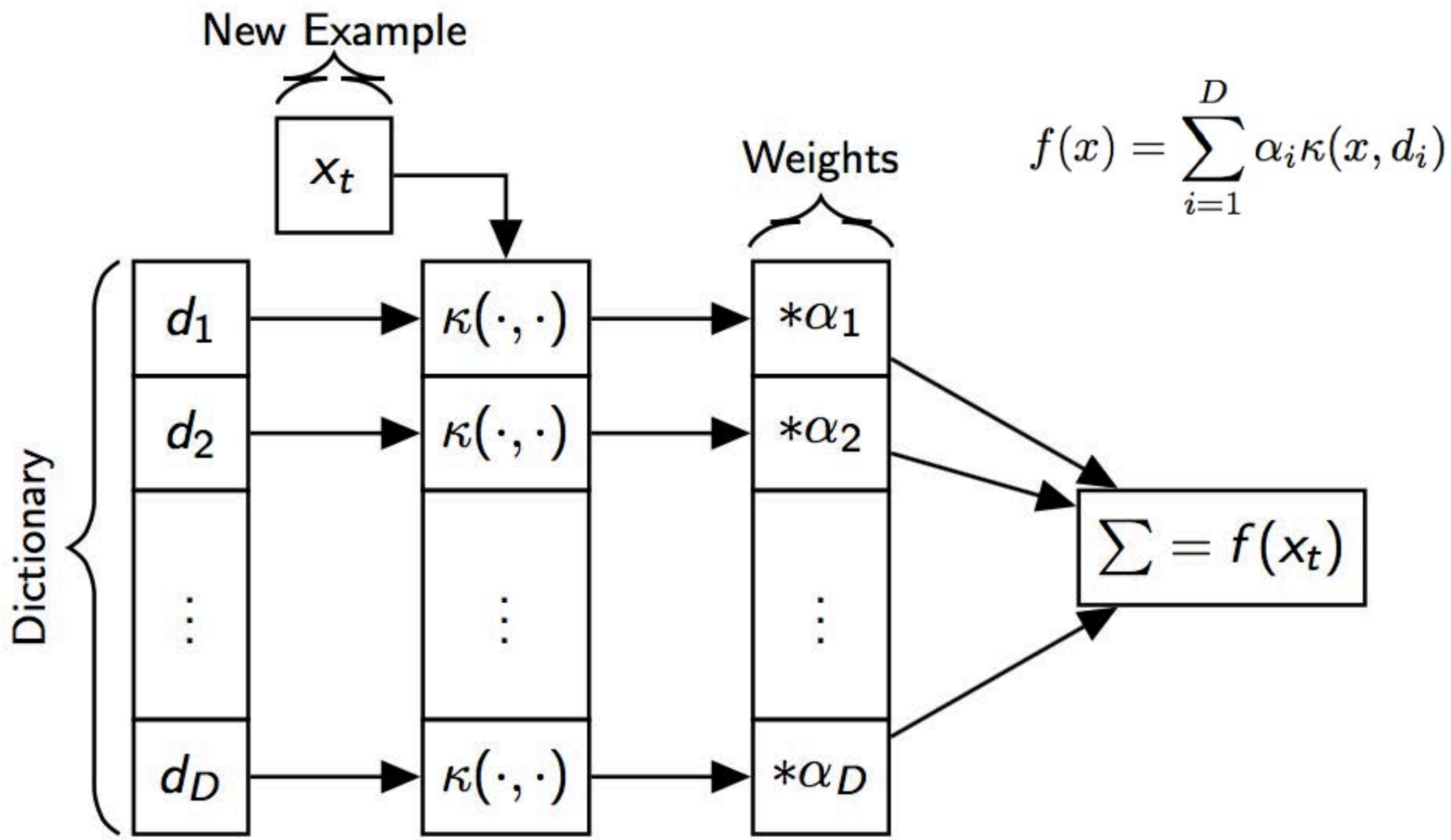
› Can be used for classification, regression, novelty detection

› Update for novelty detection

$$(\alpha_i, \alpha_t, \rho) = \begin{cases} (\Omega\alpha_i, 0, \rho + \eta\nu) \text{ if } f(x_t) \geq \rho \\ (\Omega\alpha_i, \eta, \rho - \eta(1-\nu)) \text{ otherwise} \end{cases}$$

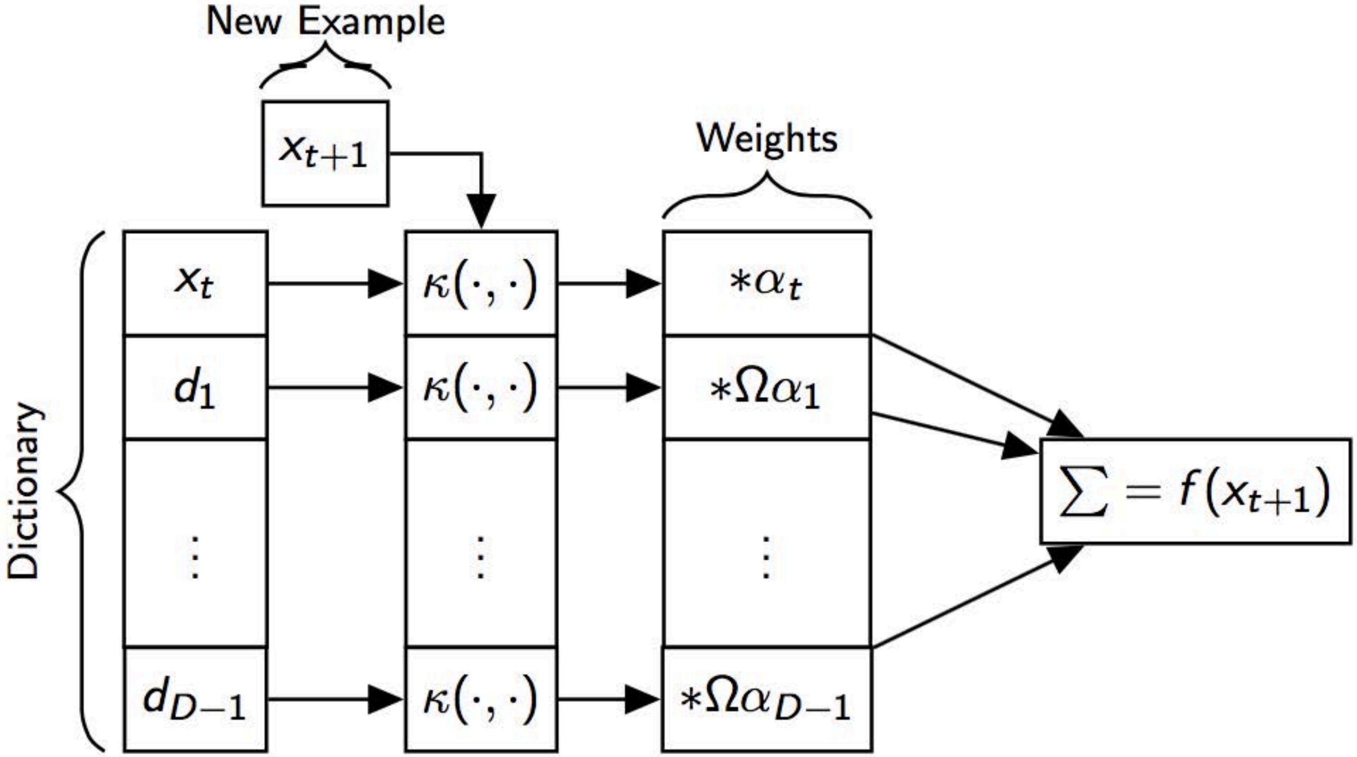
Add x_{t+1} to dictionary

Datapath for NORMA



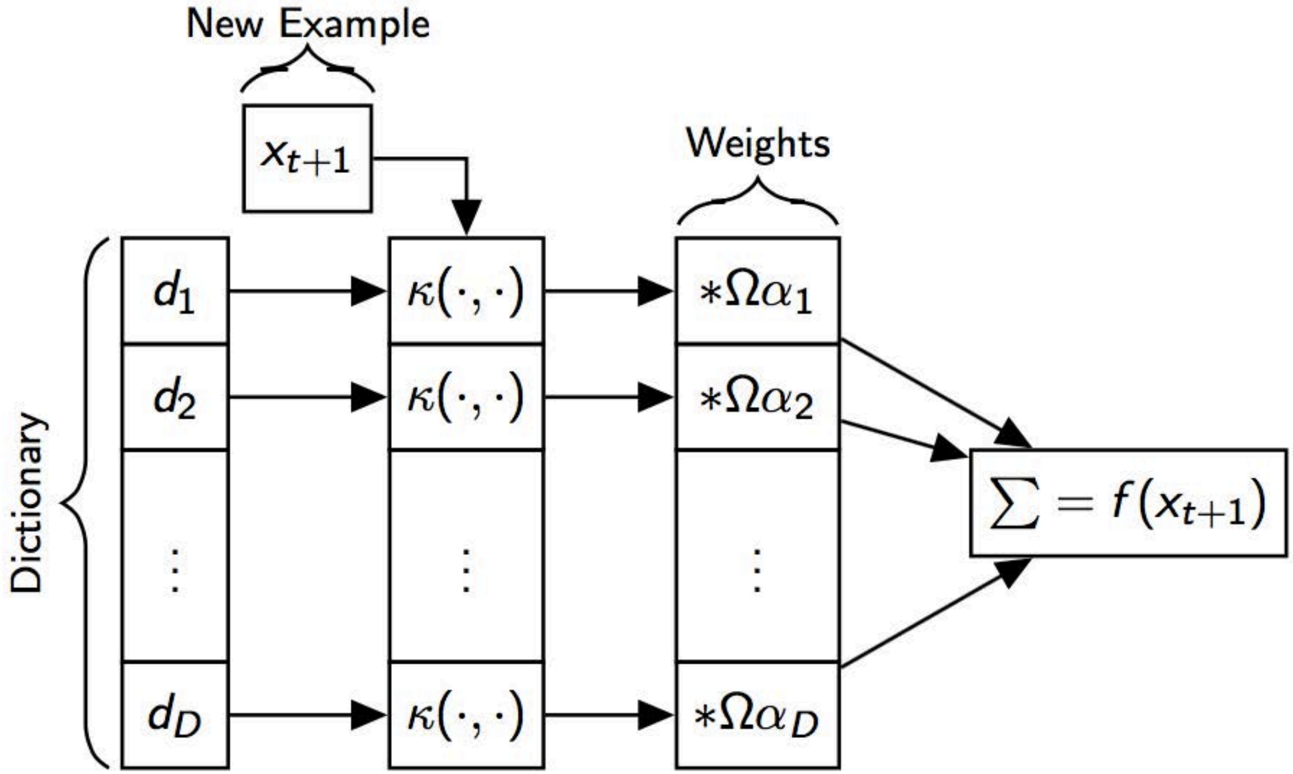
NORMA Update (Case 1)

$$(\alpha_i, \alpha_t, \rho) = \begin{cases} (\Omega\alpha_i, 0, \rho + \eta\nu) \text{ if } f(x_t) \geq \rho & \textbf{Add } \mathbf{x_{t+1}} \textbf{ to dictionary} \\ (\Omega\alpha_i, \eta, \rho - \eta(1-\nu)) \text{ otherwise} \end{cases}$$



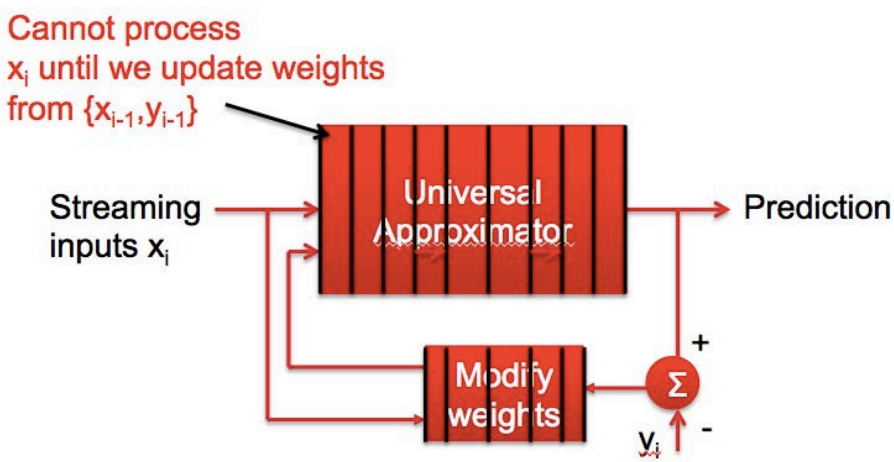
NORMA Update (Case 2)

$$(\alpha_i, \alpha_t, \rho) = \begin{cases} (\Omega\alpha_i, 0, \rho + \eta\nu) \text{ if } f(x_t) \geq \rho & \textbf{Add } \mathbf{x_{t+1}} \textbf{ to dictionary} \\ (\Omega\alpha_i, \eta, \rho - \eta(1-\nu)) \text{ otherwise} \end{cases}$$



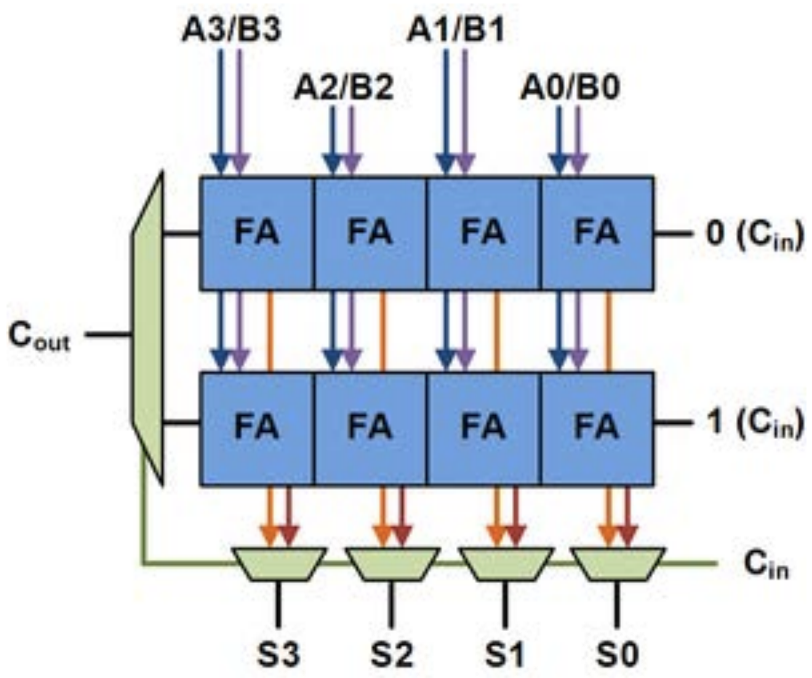
Properties of NORMA

- NORMA is a sliding window algorithm
  - If new dictionary entry added $[d_1,\cdots d_D] \rightarrow [x_t,d_1,\cdots d_{D-1}]$
  - Weight update is just a decay $\alpha_i \rightarrow \Omega\alpha_i$
  - Update cost is small compared to computing $f(x_t)$
- **Is this really true?**



Idea

› Recall carry select adder
  - implement both cases in parallel and select output



Braiding

$$f(x_{t+1}) = \sum_{i=1}^{D} \alpha_i \kappa(x_{t+1}, d_i)$$

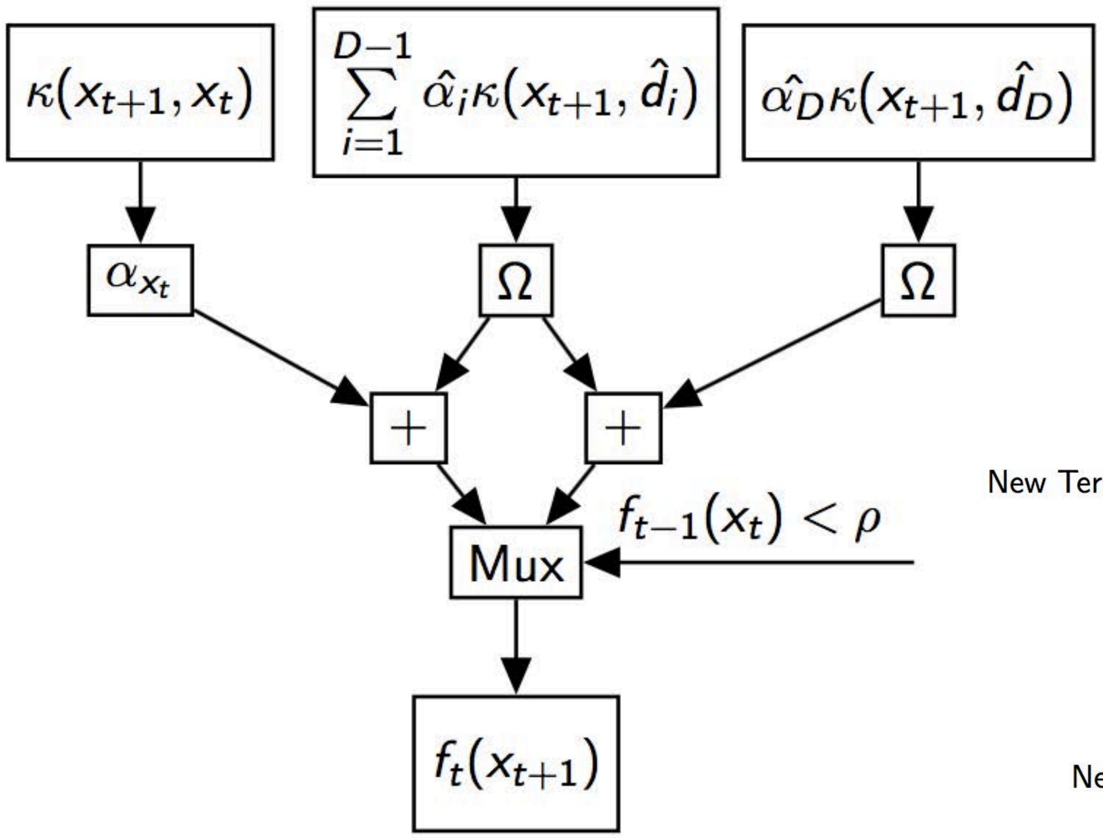
Use the previous dictionary for x_t denoted $\hat{d}_i$

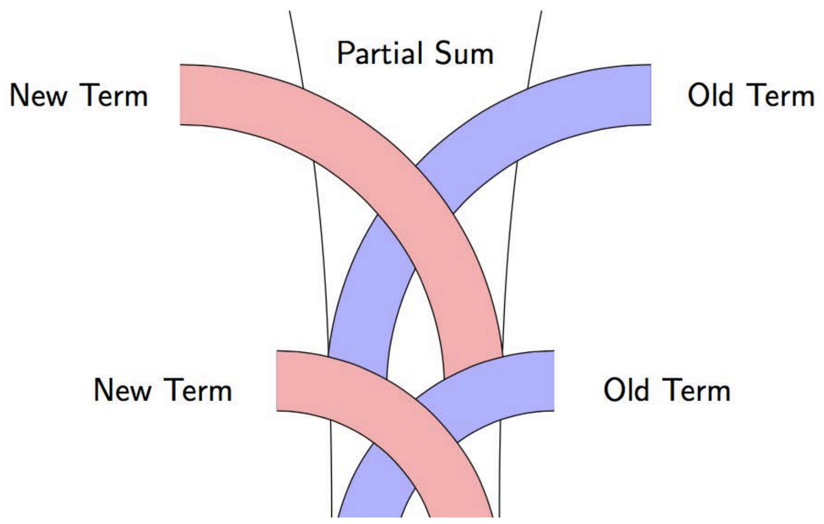
$$f(x_{t+1}) = \sum_{i=1}^{D-1} \Omega \hat{\alpha}_i \kappa(x_{t+1}, \hat{d}_i) + \text{something}$$

if x_t is added then this term $= \alpha_{x_t} \kappa(x_{t+1}, x_t)$

if x_t is not added then this term $= \Omega \hat{\alpha_D} \kappa(x_{t+1}, \hat{d}_D)$

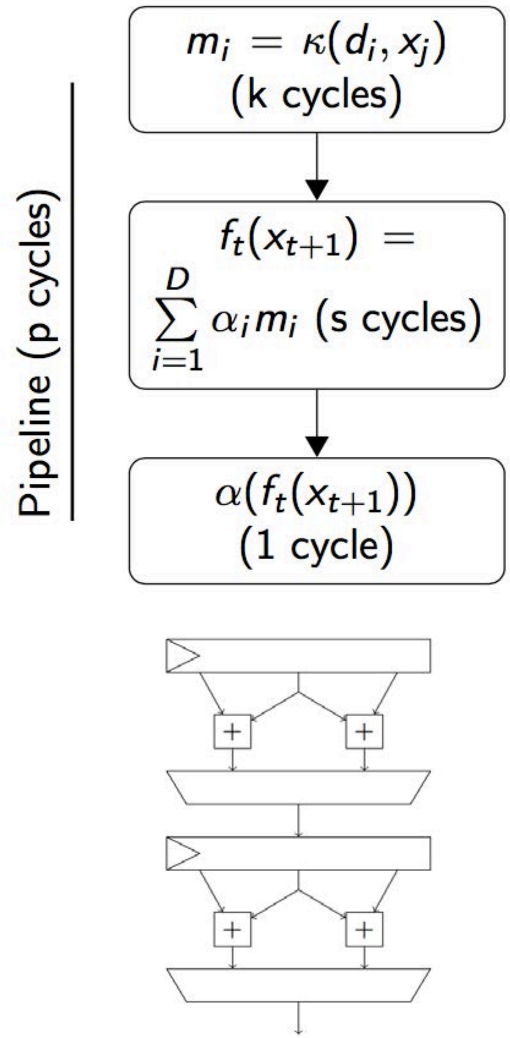
Braiding Datapath





Generalised to p cycles

$$
\begin{aligned}
f_t(x_{t+1}) = \sum_{i=1}^{D-p} \Omega^p \hat{\alpha}_i \kappa(x_{t+1}, \hat{d}_i) \\
q \left\{ \begin{array}{l}
+ \begin{cases} 0 \text{ if } x_{t+1-p} \text{ is not added} \\ \Omega^{p-1} \alpha_{x_{t+1-p}} \kappa(x_{t+1}, x_{t+1-p}) \text{ otherwise} \end{cases} \\
+ \begin{cases} 0 \text{ if } x_{t+2-p} \text{ is not added} \\ \Omega^{p-2} \alpha_{x_{t+2-p}} \kappa(x_{t+1}, x_{t+2-p}) \text{ otherwise} \end{cases} \\
\qquad \vdots \\
+ \begin{cases} 0 \text{ if } x_{t} \text{ is not added} \\ \alpha_{x_t} \kappa(x_{t+1}, x_t) \text{ otherwise} \end{cases}
\end{array} \right. \\
+ \sum_{i=D-p+1}^{D-q} \Omega^p \hat{\alpha}_i \kappa(x_{t+1}, \hat{d}_i)
\end{aligned}
$$



Implementation

- Implemented in Chisel
- On XC7VX485T- 2FFG1761C achieves ~133 MHz
- Area O(FDB^2) (F=dimensionality of input vector), time complexity O(FD)
- Speedup 500x compared with single core CPU i7-4510U (8.10 fixed)

F=8, D=	16	32	64	128	200
Frequency (MHz)	133	138	137	131	127
DSPs (/2,800)	309	514	911	1,679	2,556
Slices (/759,000)	4615	8194	14,663	29,113	46,443
Latency (cycles)	10	11	12	12	13
Speedup (×)	47	91	178	344	509
Latency reduction (×)	4.69	8.30	14.9	28.7	39.2

Comparison of Architectures

› Core with input vector F=8 and dictionary size D=16

Design	Precision	Freq MHz	Latency Cycles	T.put Cycles	Latency nS	T.put nS
Vector KNLMS	Single	282	479	479	1,699	1,699
Pipelined KNLMS	Single	314	207	1	659	3.2
Braided NORMA	8.10	113	10	1	89	8.8

Open source (GPLv2): github.com/da-steve101/chisel-pipelined-olk

Overview

- Motivation
- Kernel methods
  - Vector processor
  - Pipelined
  - Braided
  - **Distributed**
- Conclusion

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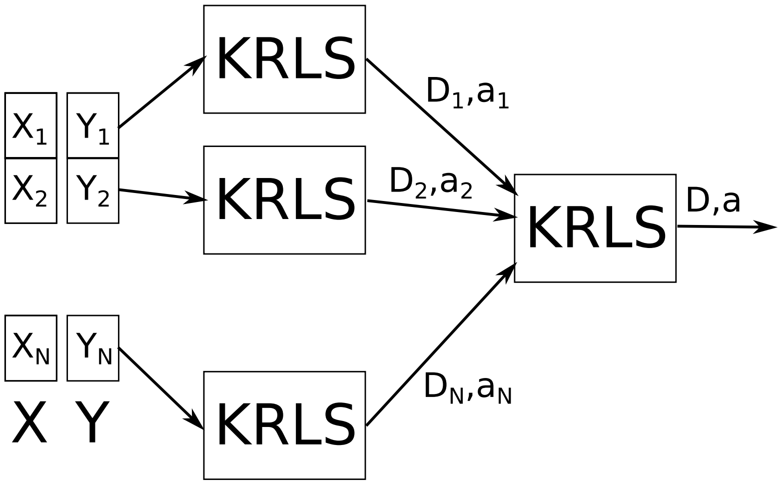
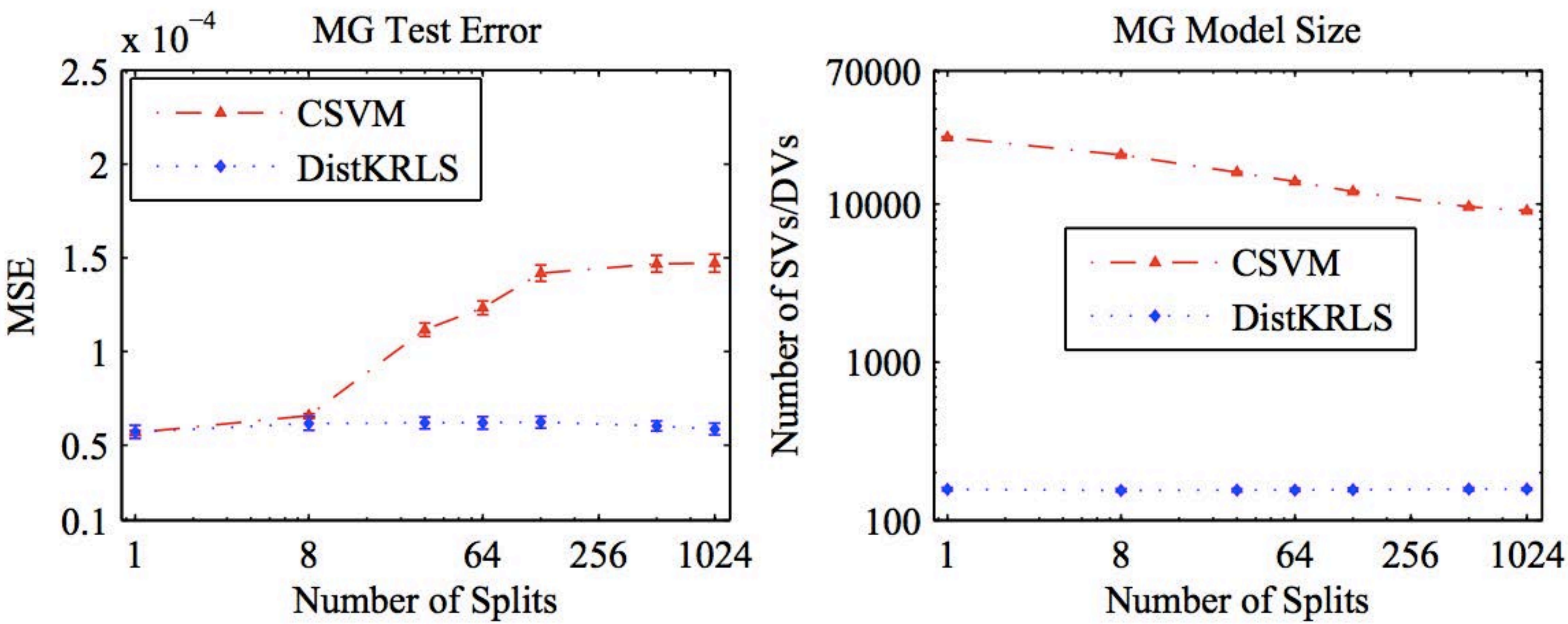
Distributed KRLS

- One problem with KRLS is how to get scalable parallelism
- Proposed a method, which uses KRLS (Engel et al. 2004) to create models on subsets of the data.
- These models can then be combined using KRLS again to create a single accurate model
  - We have shown an upper bound on the error introduced



Distributed KRLS Vs Cascade SVM

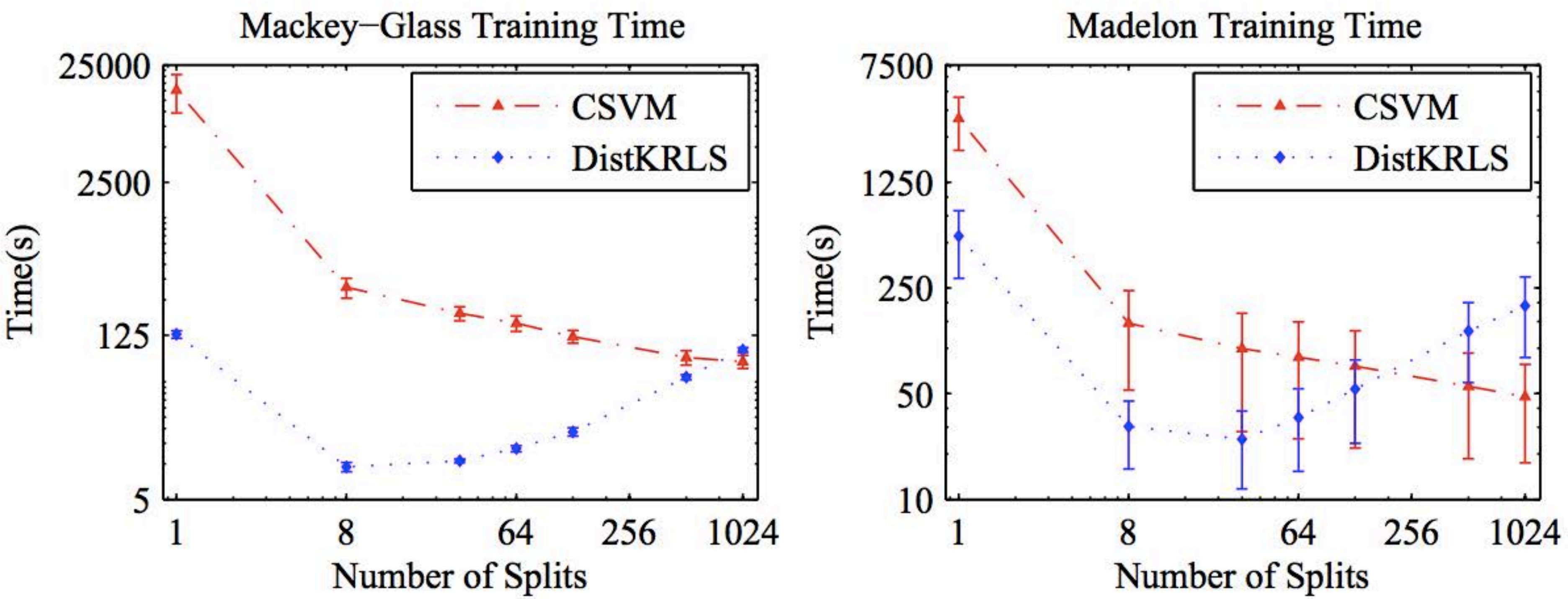
› Accuracy comparison



Speedup

Distributed KRLS Vs Cascade SVM

- Average Speedup about 20x on a 16 node cluster



Overview

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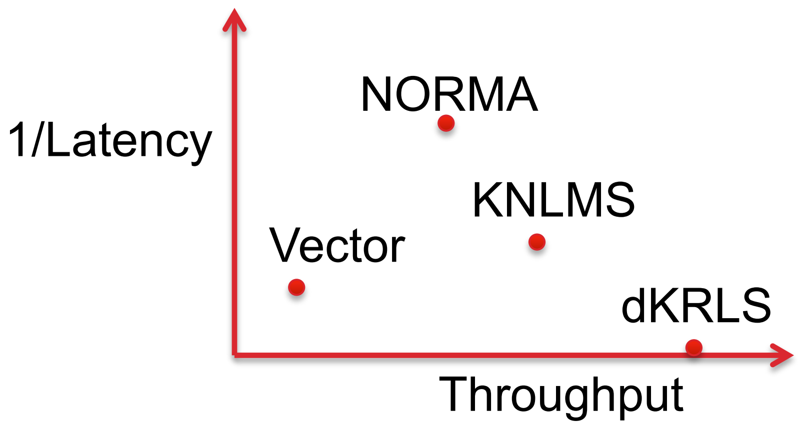
- Conclusion

Conclusion

› Demonstrated high-performance applications in ML



› Machines of the future will need to interpret and process data using ML

- FPGAs are a key enabling technology for energy-efficient, fast implementations
- A lot more to do!

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References

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